Institute of Electric Power Systems
Electric Power Engineering Section
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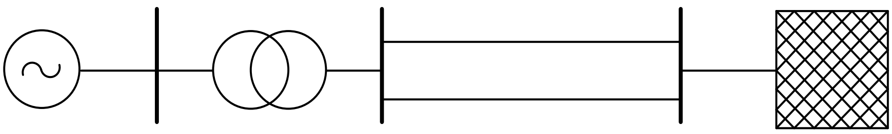
Formulary
Electric Power Engineering

Lutz Hofmann



7th Edition, September 2022

Preface

This formulary includes definitions and formulas from lectures of the *Electric Power Engineering Section* of the *Institute of Electric Power Systems (IfES)* at Leibniz University Hanover, Germany.
The formulary introduces basic mathematical knowledge and definitions as well as elementary definitions of electric power engineering using typical German symbols, e.g. for complex rotating and stationary phasors, the passive sign convention or multi-port representation of electrical devices. From the voltage equations of the symmetrical three-phase power system, the single-phase equivalent circuit diagram is derived using the symmetry conditions. The presentation of the methodology with the symmetrical components enables the reader to easily calculate unbalanced operating states as a result of, e.g., unbalanced short circuits or interruptions by a fault-specific interconnection of the positive-, negative- and zero-sequence systems.
For the system-defining equipment of electrical power systems, such as synchronous machines, induction machines, equivalent networks, transformers and lines, the respective positive-, negative- and zero-sequence equivalent circuits for the calculation of the steady-state operating behavior are presented and, if necessary, supplemented by equivalent circuits for the transient and subtransient operating behavior.
Finally, basic methods for network calculation, equipment design and network control are presented, such as a method for the calculation of current distributions in medium- and low-voltage networks, stability analysis of the single machine problem, frequency control based on the dynamic balance model as well as short-circuit current calculation, especially according to IEC EN 60909. Furthermore, the calculation of line-to-ground faults in networks with different types of star point grounding is presented.

The authors hope that this collection of formulae will not only serve students as an assistance in their exams and lecture-accompanying exercises but will also be of use in their future professional life.

This is the seventh edition of the formulary provided by the Electric Power Engineering Section in English language. Suggestions for this edition are welcome and can be submitted to hofmann@ifes.uni-hannover.de.

Hannover, September 2022

Lutz Hofmann and the research assistants of the
Electric Power Engineering Section of IfES

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1 Symbols and Abbreviations

General Symbols

g	Instantaneous value	$\boldsymbol{A}$	Matrix
$\hat{g}$	Value of an amplitude	$\boldsymbol{a}$	(Column)vector
G	RMS value	j	Imaginary unit

General Explanations

Electric quantities are given as RMS values.	U, I
Equations describing single-phase equivalent circuit diagrams consist of phase quantities.	U, I
Rated quantities are marked by the index r.	U_r
Nominal and rated voltages (index n or r) are line-to-line voltages.	U_n, U_r
Time-dependent quantities are written in lower-case letters.	u, i
Complex quantities are indicated by underlining.	$\underline{U}, \underline{I}$

Nomenclature According to DIN EN 60027-7:2011

Position	Meaning	Example	Explanation
0	Identification	U_q, X_σ, I_r	Voltage source, leakage reactance, rated current
1	Phase quantities a, b, c or symmetrical components 1, 2, 0	$\underline{U}_1$	Voltage of the positive-sequence system
2	Operating state	$\underline{U}_{1k1}$	Phase-to-ground fault
3	Electrical device	$\underline{U}_{1k1T4}$	At the transformer T4
4	Location	$\underline{U}_{1k1T4OS}$	At the high-voltage side
5	Additional information	$\underline{U}_{1k1T4OSmax}$	Highest value

Note:

For the designation according to DIN EN 60027-7:2011, specific preferred indexes are chosen for position 0), for example:

Source voltage in the positive-sequence system: U_{q1}

Source voltage in phase a: U_{qa}

Symbols

a	Distance	R	Resistance
A	Cross-sectional area, energy yield	s	Slip, proportional droop
		S	Apparent power
B	Susceptance, magnetic flux density	t	Time
		T	Period
c	Voltage factor	U	Voltage
C	Capacitance	v	Detuning, level of intermeshing
d	Diameter, attenuation		
E	Energy	w	Number of turns
F	Force	$\ddot{u}$	Transformation ratio
G	Conductance	Z	Impedance, section modulus
H	Magnetic field strength	γ	Propagation constant
I	Current	δ	Rotor displacement angle ground fault factor
k	Proportional gain, imbalance factor		
		ε	Field ratio
K	Correction factor	η	Efficiency
l	Length	ρ_{E}	Specific ground resistance
L	Inductance		
n	Rotational speed	φ	Phase angle
p	Number of pole pairs	σ	Mechanical stress
P	Active power	ω	Angular frequency
Q	Reactive power, heat	Ω	Mechanical angular frequency

Abbreviations

ASC	Active sign convention	kap./cap.	Capacitive
E	Ground potential	M	Neutral point, star point
ESB	Equivalent circuit	PSC	Passive sign convention
ind.	Inductive	SC	Symmetrical components

Special symbols and mathematical operations

$\underline{a} = \mathrm{e}^{\mathrm{j}2\pi/3}$	Complex operator of length 1 (unit phasor)	$\lvert\underline{G}\rvert = G$	Absolute value of $\underline{G}$
Δ	Difference	$\mathrm{Re}\{\underline{G}\} = G_{\perp}$	Real part of $\underline{G}$
		$\mathrm{Im}\{\underline{G}\} = G_{\perp\perp}$	Imaginary part of $\underline{G}$

Examples of per unit and per unit length quantities

$\underline{z} = \dfrac{\underline{Z}}{\underline{Z}_{\mathrm{B}}}$ — $\underline{z}$ in relation to $\underline{Z}_{\mathrm{B}}$ in p.u. or in %

$\underline{Z}' = \dfrac{\underline{Z}}{l}$ — Impedance per unit length $\underline{Z}'$ in Ω/m or Ω/km

Superscript indices

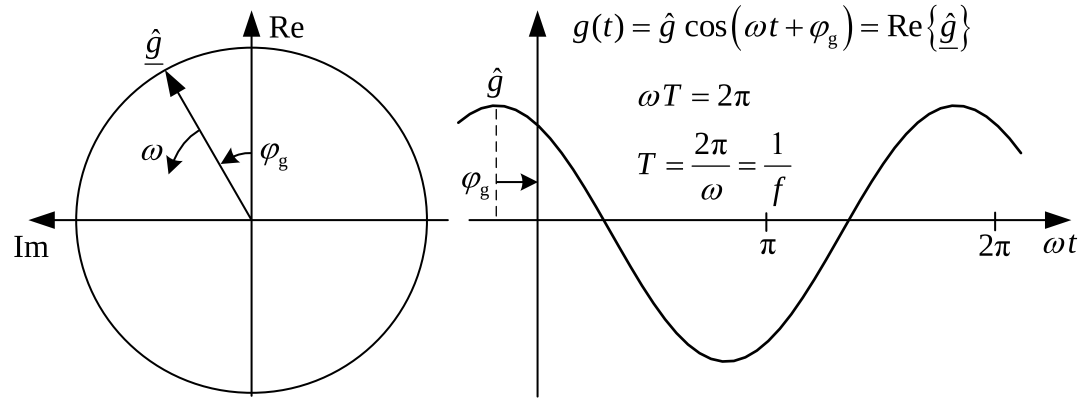
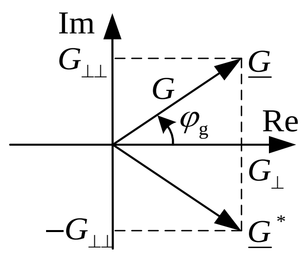
'	Transient parameter (voltage, reactance, etc.) or parameter converted to another voltage level or to the stator side or a per unit length parameter
"	Subtransient parameter (voltage, reactance, etc.)
*	Conjugate parameter
T	Transposed matrix/vector

Subscript indices (a selection) that describe individual variables in more detail

Δ	Delta connection parameter
Y	Wye connection parameter
1, 2, 0	Positive- (1), negative- (2) and zero- (0)sequence system
0	Synchronous, steady-state operating point
a, b, c	Phase a, phase b, phase c
ab	Emitted or output
b	Reactive portion
B	Reference value
C	Capacitive, charging
d	d-axis
D	Damper longitudinal axis, attenuation
el	Electric
ers	Equivalent, substitute
erz	Generation
f	Field (excitation)
F	Fault
g	Mutual inductive or capacitive coupling
G	Generator
h, i, j, ν	Counting index
i	Internal parameter
inst	Installed (power)
K	Correction
k	Short-circuit
k, k3	3-ph. line-to-ground fault
k1	1-ph. line-to-ground fault
k2	2-ph. line-to-line fault
k2E	2-ph. line-to-ground fault
kin	Kinetic
ℓ, l	Open-circuit operation, no-load operation
L	(Transmission) line
m	Magnetizing, mechanical, main conductor
max	Maximum
min	Minimum
M	Motor, neutral-point, machine-
ME	Neutral-to-ground
MV	Medium-voltage winding (three-winding transformer)
n	Nominal
N	(External) grid
HV	High-voltage winding (transformer)
q	Source, q-(axis) parameter
Q	Damper quadrature axis, source
r	Rated
rel	per unit
s	Self, stator, sub-conductor
S	Symmetrical components
SG	Vector group
Str	Phase
T	Turbine, transformer
LV	Low-voltage winding (transformer)
V	Losses
w	Active portion, resisting
W	Characteristic (impedance)
zu	absorbed, input

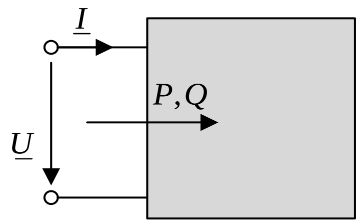
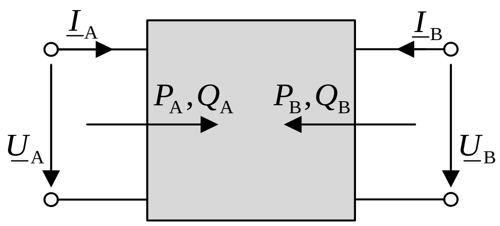
2 Fundamentals

2.1 Instantaneous Values and Phasor Representation

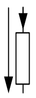
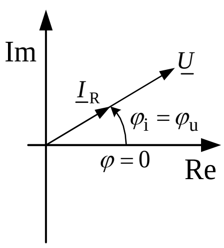
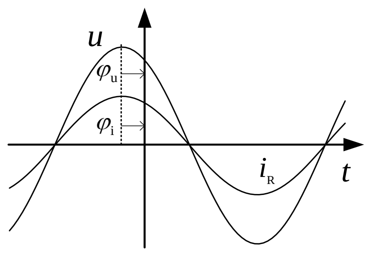
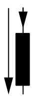
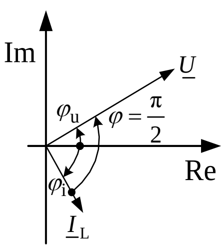
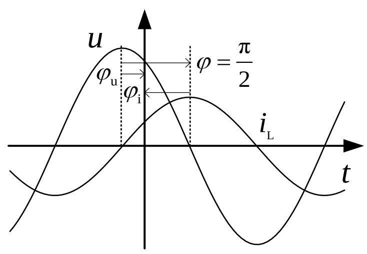
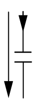
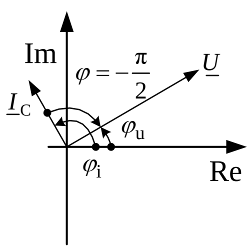
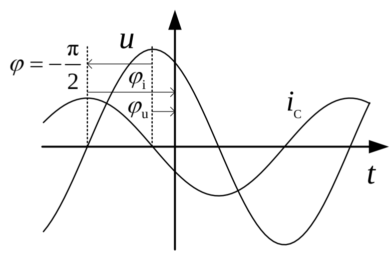
Rotating amplitude phasor $\underline{\hat{g}}$	
Definition of rotating phasor	$\underline{\hat{g}} = \hat{g}\,(\cos(\omega t + \varphi_{\mathrm{g}}) + \mathrm{j}\sin(\omega t + \varphi_{\mathrm{g}}))$ $= \hat{g}\,\mathrm{e}^{\mathrm{j}(\omega t + \varphi_{\mathrm{g}})} = \hat{g}\,\mathrm{e}^{\mathrm{j}\omega t}\mathrm{e}^{\mathrm{j}\varphi_{\mathrm{g}}} = \mathrm{Re}\{\underline{\hat{g}}\} + \mathrm{j}\,\mathrm{Im}\{\underline{\hat{g}}\}$
Conjugate rotating phasor	$\underline{\hat{g}}^{*} = \hat{g}\,\mathrm{e}^{-\mathrm{j}(\omega t + \varphi_{\mathrm{g}})}$
Correlation between instantaneous values and rotating phasor	
	
Initial phase	φ_{g} (Phase angle at $t = 0$)
Stationary phasor $\underline{G}$ [1)]	
Phasor	$\underline{G} = \frac{\underline{\hat{g}}}{\sqrt{2}\,\mathrm{e}^{\mathrm{j}\omega t}} = G\,\mathrm{e}^{\mathrm{j}\varphi_{\mathrm{g}}}$ $= G\,(\cos\varphi_{\mathrm{g}} + \mathrm{j}\sin\varphi_{\mathrm{g}})$ $= \mathrm{Re}(\underline{G}) + \mathrm{j}\,\mathrm{Im}(\underline{G}) = G_{\perp} + \mathrm{j}G_{\perp\perp}$
Conjugate phasor	$\underline{G}^{*} = G\,\mathrm{e}^{-\mathrm{j}\varphi_{\mathrm{g}}} = G\left(\cos\varphi_{\mathrm{g}} - \mathrm{j}\sin\varphi_{\mathrm{g}}\right) = G_{\perp} - \mathrm{j}G_{\perp\perp}$
Phasor diagram	

1) Stationary phasors are hereinafter called "phasors".

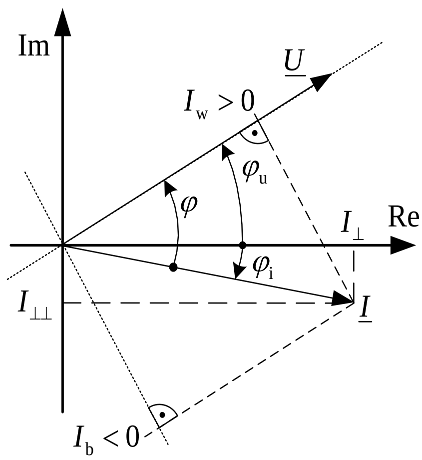
2.2 Passive Sign Convention (PSC) and Reference Direction of the Active and Reactive Power

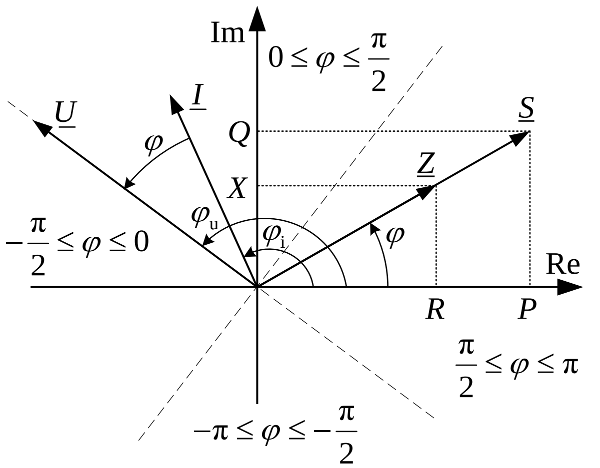
<table>
<tr><th colspan="2">PSC: Nominal direction convention for terminal voltage and current of an element</th></tr>
<tr><td>Voltage, current, active/reactive power arrows point in the same direction for a two-terminal element in the PSC</td><td></td></tr>
<tr><td>Voltage, current, active/reactive power arrows point in the same direction at each side for a four-terminal element in the PSC</td><td></td></tr>
</table>

2.3 Relations Between Sinusoidal Voltages and Currents in the Time and Frequency Domain for Linear Elements

<table>
<tr><th></th><th>$u(t) = \hat{u}\cos(\omega t + \varphi_\mathrm{u})$</th><th>Phasor diagram</th><th>Voltage and current as a function of time</th></tr>
<tr><td>R
</td><td>$i_\mathrm{R}(t) = \frac{u(t)}{R}$
$= \frac{\hat{u}}{R}\cos(\omega t + \varphi_\mathrm{u})$
$= \hat{i}_\mathrm{R}\cos(\omega t + \varphi_\mathrm{i})$</td><td></td><td></td></tr>
<tr><td>L
</td><td>$i_\mathrm{L}(t) = \frac{1}{L}\int u(t)\,\mathrm{d}t$
$= \frac{\hat{u}}{\omega L}\sin(\omega t + \varphi_\mathrm{u})$
$= \hat{i}_\mathrm{L}\cos(\omega t + \varphi_\mathrm{u} - \frac{\pi}{2})$
$= \hat{i}_\mathrm{L}\cos(\omega t + \varphi_\mathrm{i})$</td><td></td><td></td></tr>
<tr><td>C
</td><td>$i_\mathrm{C}(t) = C\frac{\mathrm{d}u(t)}{\mathrm{d}t}$
$= -\hat{u}\omega C\sin(\omega t + \varphi_\mathrm{u})$
$= \hat{i}_\mathrm{C}\cos(\omega t + \varphi_\mathrm{u} + \frac{\pi}{2})$
$= \hat{i}_\mathrm{C}\cos(\omega t + \varphi_\mathrm{i})$</td><td></td><td></td></tr>
<tr><td colspan="4">Phase shift $\varphi = \varphi_\mathrm{u} - \varphi_\mathrm{i}$ (Pointing from current to voltage)</td></tr>
</table>

2.4 Power in AC Circuits Using the PSC

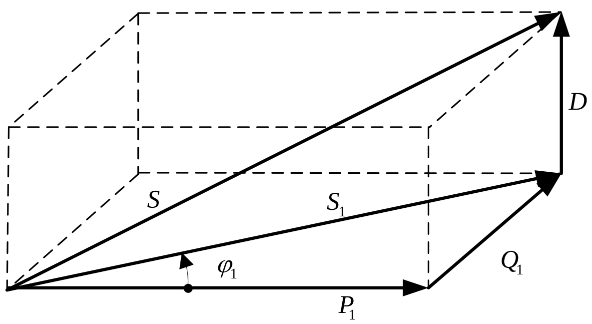
Instantaneous power $p(t)$	
Instantaneous power of a two-terminal (one-port) element with terminal voltage $u(t)$ and terminal current $i(t)$	$\begin{aligned} p(t) &= u(t)\cdot i(t) \\ &= U\,I\left(\cos\varphi + \cos\left(2\omega t + \varphi_\mathrm{U} + \varphi_\mathrm{I}\right)\right) \\ &= P + S\cos\left(2\omega t + \varphi_\mathrm{U} + \varphi_\mathrm{I}\right) \\ &= P\left(1+\cos\left(2\omega t + \varphi_\mathrm{U}\right)\right) + Q\sin\left(2\omega t + \varphi_\mathrm{U}\right) \\ &= p_\mathrm{P}(t) + p_\mathrm{Q}(t) \end{aligned}$
Complex apparent power $\underline{S}$, active power P and reactive power Q	
Power equation of a two-terminal (one-port) element with terminal voltage $\underline{U}$ and terminal current $\underline{I}$	$\underline{S} = \underline{U}\,\underline{I}^* = U\,I\,\mathrm{e}^{\mathrm{j}(\varphi_\mathrm{u}-\varphi_\mathrm{i})} = U\,I\,\mathrm{e}^{\mathrm{j}\varphi} = S\,\mathrm{e}^{\mathrm{j}\varphi}$ $= P + \mathrm{j}Q = S(\cos\varphi + \mathrm{j}\sin\varphi)$
Active and reactive current	$\underline{I} = \dfrac{\underline{S}^*}{\underline{U}^*} = \dfrac{P - \mathrm{j}Q}{U}\,\mathrm{e}^{\mathrm{j}\varphi_\mathrm{u}} = (I_\mathrm{w} + \mathrm{j}\,I_\mathrm{b})\,\mathrm{e}^{\mathrm{j}\varphi_\mathrm{u}}$ $I = \sqrt{I_\mathrm{w}^2 + I_\mathrm{b}^2}$
Phasor diagram	
Relation between active/reactive power and active/reactive currents	$P = U\,I_\mathrm{w} = U\,I\cos\varphi$ $Q = -U\,I_\mathrm{b} = -U\,I\sin\varphi$
Active/displacement factor	$-1 \le \cos\varphi \le 1$

<table>
<tr><td colspan="3">Power conventions in the PSC
(equal reference direction convention as in Section 2.2)</td><td>$P>0$ → Active power consumption
(consumer)
$P<0$ → Active power output
(producer)
$Q>0$ → Reactive power consumption
(inductive behavior)
$Q<0$ → Reactive power output
(capacitive behavior)</td></tr>
<tr><td>$0 \le \varphi \le \frac{\pi}{2}$</td><td>$P>0$</td><td>$Q>0$</td><td rowspan="4">
</td></tr>
<tr><td>$-\frac{\pi}{2} \le \varphi \le 0$</td><td>$P>0$</td><td>$Q<0$</td></tr>
<tr><td>$-\pi \le \varphi \le -\frac{\pi}{2}$</td><td>$P<0$</td><td>$Q<0$</td></tr>
<tr><td>$\frac{\pi}{2} \le \varphi \le \pi$</td><td>$P<0$</td><td>$Q>0$</td></tr>
</table>

2.5 Impedance, Admittance and Apparent Power of Basic Elements Using the PSC

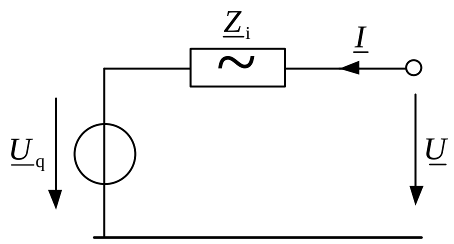
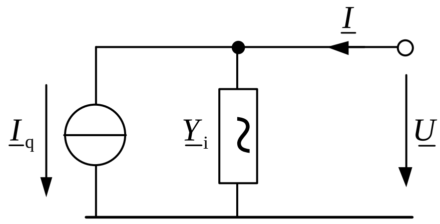
<table>
<tr><td colspan="4">Impedance $\underline{Z}$ (with resistance R and reactance X)</td></tr>
<tr><td colspan="4">$\underline{Z} = (+)\frac{\underline{U}}{\underline{I}} = \frac{U}{I}\,\mathrm{e}^{\mathrm{j}(\varphi_u - \varphi_i)} = Z\,\mathrm{e}^{\mathrm{j}\varphi_Z} = \mathrm{Re}(\underline{Z}) + \mathrm{j}\,\mathrm{Im}(\underline{Z}) = Z\,(\cos\varphi_Z + \mathrm{j}\sin\varphi_Z) = R + \mathrm{j}\,X$</td></tr>
<tr><td colspan="4">$Z = \sqrt{R^2 + X^2}$ and $\varphi_Z = \varphi_u - \varphi_i = \arctan\frac{X}{R}$ (for $R>0$)</td></tr>
<tr><td colspan="4">Admittance $\underline{Y}$ (with conductance G and susceptance B)</td></tr>
<tr><td colspan="4">$\underline{Y} = (+)\frac{\underline{I}}{\underline{U}} = \frac{I}{U}\,\mathrm{e}^{\mathrm{j}(\varphi_i - \varphi_u)} = Y\,\mathrm{e}^{\mathrm{j}\varphi_Y} = \mathrm{Re}(\underline{Y}) + \mathrm{j}\,\mathrm{Im}(\underline{Y}) = Y\,(\cos\varphi_Y + \mathrm{j}\sin\varphi_Y) = G + \mathrm{j}\,B$</td></tr>
<tr><td colspan="4">$Y = \sqrt{G^2 + B^2}$ and $\varphi_Y = -\varphi_Z = \varphi_i - \varphi_u = \arctan\frac{B}{G}$ (for $G>0$)</td></tr>
<tr><td>Element</td><td>$\underline{Z} = R + \mathrm{j}\,X$</td><td>$\underline{Y} = G + \mathrm{j}\,B$</td><td>$\underline{S} = P + \mathrm{j}Q = \underline{U}\,\underline{I}^*$</td></tr>
<tr><td>Resistor</td><td>R</td><td>$\frac{1}{R} = G$</td><td>$R\,I^2 = G\,U^2 = U\,I_w$</td></tr>
<tr><td>Inductor</td><td>$\mathrm{j}\omega L = \mathrm{j}\,X_L$</td><td>$\frac{1}{\mathrm{j}\omega L} = -\mathrm{j}\,B_L$</td><td>$\mathrm{j}\,X_L\,I^2 = \mathrm{j}\,B_L\,U^2 = -\mathrm{j}U\,I_b$</td></tr>
<tr><td>Capacitor</td><td>$\frac{1}{\mathrm{j}\omega C} = -\mathrm{j}\,X_C$</td><td>$\mathrm{j}\omega C = \mathrm{j}\,B_C$</td><td>$-\mathrm{j}\,X_C\,I^2 = -\mathrm{j}\,B_C\,U^2 = -\mathrm{j}U\,I_b$</td></tr>
</table>

2.6 Harmonics

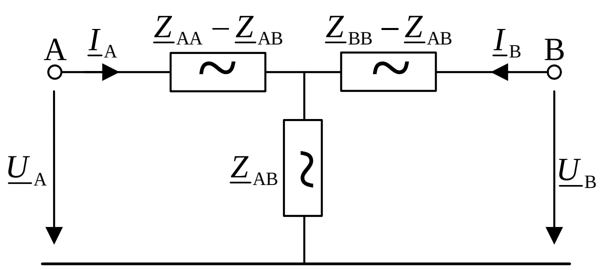
Harmonics	
RMS value of a signal subjected to harmonics	$G=\sqrt{G_1^2+G_2^2+\ldots+G_\infty^2}=\sqrt{\sum_{h=1}^{\infty}G_h^2}$
h = 1: Fundamental component with frequency f_1 h > 1: Order number of harmonic with frequency $f_h = h\cdot f_1$	
Fundamental factor	$g=\frac{G_1}{\sqrt{G_1^2+G_2^2+\ldots+G_\infty^2}}=\frac{G_1}{G}$
Total harmonic distortion (THD)	$d=\frac{\sqrt{G_2^2+\ldots+G_\infty^2}}{\sqrt{G_1^2+G_2^2+\ldots+G_\infty^2}}=\frac{\sqrt{G_2^2+\ldots+G_\infty^2}}{G}$
Relation between g and d	$g^2+d^2=1$
Apparent power S, fundamental apparent power S_1 and distortion power D	$S^2=U^2I^2=\sum_{h=1}^{\infty}U_h^2\sum_{h=1}^{\infty}I_h^2$ $=g_U^2g_I^2U^2I^2+\left(g_U^2d_I^2+g_I^2d_U^2+d_U^2d_I^2\right)U^2I^2$ $=S_1^2+D^2=P_1^2+Q_1^2+D^2$
Fundamental active/reactive power	$\underline{S}_1=\underline{U}_1\underline{I}_1^*=P_1+\mathrm{j}Q_1=U_1I_1\left(\cos\varphi_1+\mathrm{j}\sin\varphi_1\right)$
Relation between apparent, active, reactive and distortion power	
Power factor (see active/displacement factor in Section 2.4)	$\lambda=\frac{\lvert P_1\rvert}{S}=\frac{U_1I_1\lvert\cos\varphi_1\rvert}{U\,I}=g_U\,g_I\lvert\cos\varphi_1\rvert\le 1$

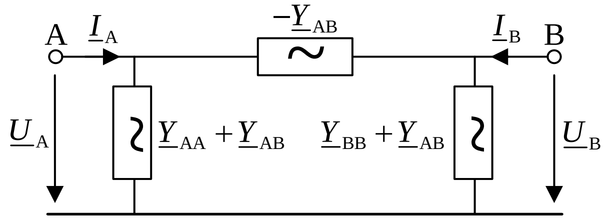
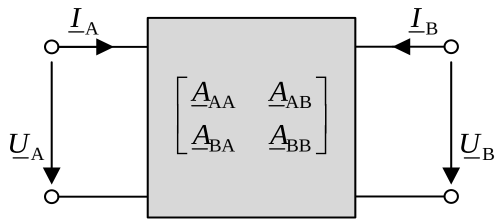
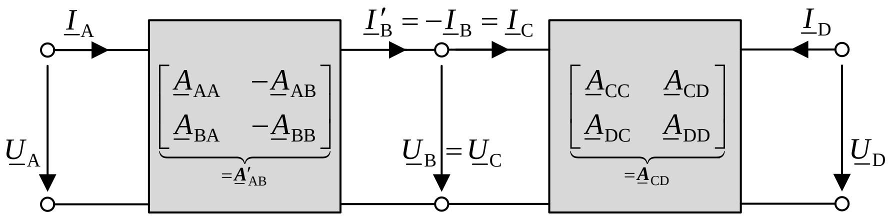
2.7 Multi-Port Theory

2.7.1 One-Port Networks

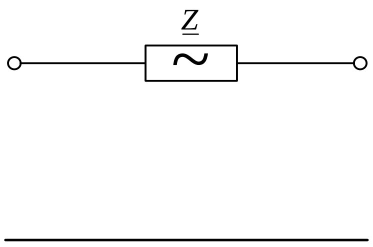
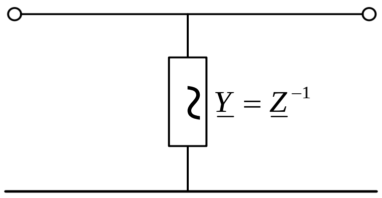
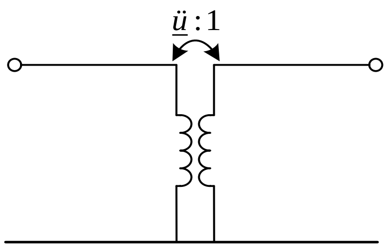
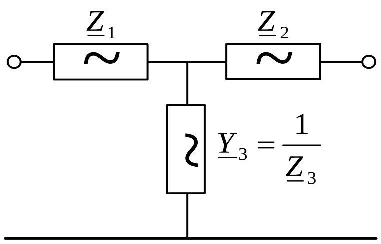
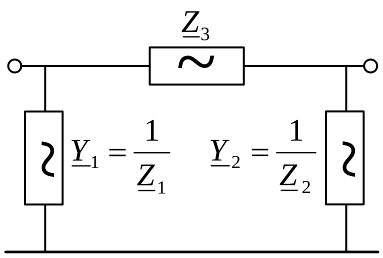
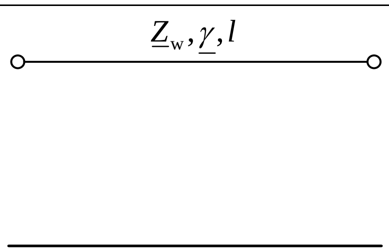
Voltage source equivalent circuit $\underline{U} = \underline{Z}_\mathrm{i}\,\underline{I} + \underline{U}_\mathrm{q}$	
Current source equivalent circuit $\underline{I} = \underline{Y}_\mathrm{i}\,\underline{U} + \underline{I}_\mathrm{q}$	
Open-circuit operation (No-load operation)	$\underline{U} = \underline{U}_\ell = \underline{U}_\mathrm{q}$ resp. $\underline{U} = \underline{U}_\ell = -\frac{1}{\underline{Y}_\mathrm{i}}\underline{I}_\mathrm{q}$
Short-circuit operation	$\underline{I} = \underline{I}_\mathrm{k} = -\frac{1}{\underline{Z}_\mathrm{i}}\underline{U}_\mathrm{q}$ resp. $\underline{I} = \underline{I}_\mathrm{k} = \underline{I}_\mathrm{q}$
Conversion (identical terminal behavior)	$\underline{Z}_\mathrm{i} = -\frac{\underline{U}_\mathrm{q}}{\underline{I}_\mathrm{q}} = -\frac{\underline{U}_\ell}{\underline{I}_\mathrm{k}}$ resp. $\underline{U}_\mathrm{q} = -\underline{Z}_i\underline{I}_\mathrm{q}$

2.7.2 Two-Port Networks

Impedance representation	
T-equivalent circuit	
Two-port equation (Z-characteristic)	$\begin{bmatrix}\underline{U}_\mathrm{A}\\ \underline{U}_\mathrm{B}\end{bmatrix} = \begin{bmatrix}\underline{Z}_\mathrm{AA} = \left.\frac{\underline{U}_\mathrm{A}}{\underline{I}_\mathrm{A}}\right\rvert_{\underline{I}_\mathrm{B}=0} & \underline{Z}_\mathrm{AB} = \left.\frac{\underline{U}_\mathrm{A}}{\underline{I}_\mathrm{B}}\right\rvert_{\underline{I}_\mathrm{A}=0}\\ \underline{Z}_\mathrm{BA} = \left.\frac{\underline{U}_\mathrm{B}}{\underline{I}_\mathrm{A}}\right\rvert_{\underline{I}_\mathrm{B}=0} & \underline{Z}_\mathrm{BB} = \left.\frac{\underline{U}_\mathrm{B}}{\underline{I}_\mathrm{B}}\right\rvert_{\underline{I}_\mathrm{A}=0}\end{bmatrix}\begin{bmatrix}\underline{I}_\mathrm{A}\\ \underline{I}_\mathrm{B}\end{bmatrix}$ $\underline{\boldsymbol{u}} = \underline{\boldsymbol{Z}}\ \underline{\boldsymbol{i}}$

Admittance representation	
Π-equivalent circuit	
Two-port equation (Y-characteristic)	$\begin{bmatrix}\underline{I}_A\\ \underline{I}_B\end{bmatrix}=\begin{bmatrix}\underline{Y}_{AA}=\left.\frac{\underline{I}_A}{\underline{U}_A}\right\vert_{\underline{U}_B=0} & \underline{Y}_{AB}=\left.\frac{\underline{I}_A}{\underline{U}_B}\right\vert_{\underline{U}_A=0}\\ \underline{Y}_{BA}=\left.\frac{\underline{I}_B}{\underline{U}_A}\right\vert_{\underline{U}_B=0} & \underline{Y}_{BB}=\left.\frac{\underline{I}_B}{\underline{U}_B}\right\vert_{\underline{U}_A=0}\end{bmatrix}\begin{bmatrix}\underline{U}_A\\ \underline{U}_B\end{bmatrix}$ $\underline{\boldsymbol{i}}=\underline{\boldsymbol{Y}}\,\underline{\boldsymbol{u}}$
Iterative representation (cascade or transmission representation)	
Iterative form	
Terminal quantities of side A in dependence of side B (transmission characteristic)	$\begin{bmatrix}\underline{U}_A\\ \underline{I}_A\end{bmatrix}=\begin{bmatrix}\underline{A}_{AA}=\left.\frac{\underline{U}_A}{\underline{U}_B}\right\vert_{\underline{I}_B=0} & \underline{A}_{AB}=\left.\frac{\underline{U}_A}{\underline{I}_B}\right\vert_{\underline{U}_B=0}\\ \underline{A}_{BA}=\left.\frac{\underline{I}_A}{\underline{U}_B}\right\vert_{\underline{I}_B=0} & \underline{A}_{BB}=\left.\frac{\underline{I}_A}{\underline{I}_B}\right\vert_{\underline{U}_B=0}\end{bmatrix}\begin{bmatrix}\underline{U}_B\\ \underline{I}_B\end{bmatrix}$ $\underline{\boldsymbol{z}}_A=\underline{\boldsymbol{A}}_{AB}\,\underline{\boldsymbol{z}}_B$
Cascade connection of two-port networks	
	
Terminal quantities of side A in dependence of side D	$\underline{\boldsymbol{z}}_A=\underline{\boldsymbol{A}}'_{AB}\,\underline{\boldsymbol{z}}'_B=\underline{\boldsymbol{A}}'_{AB}\underline{\boldsymbol{A}}_{CD}\,\underline{\boldsymbol{z}}_D$ For the cascaded connection, the terminal values of side B are expressed in the ASC (primed variables)
Inversion	
Inverse of a 2×2 matrix ($\det(\boldsymbol{A})=A_{AA}A_{BB}-A_{AB}A_{BA}$)	$\boldsymbol{A}^{-1}=\frac{\operatorname{adj}(\boldsymbol{A})}{\det(\boldsymbol{A})}=\frac{1}{\det(\boldsymbol{A})}\cdot\begin{bmatrix}A_{BB} & -A_{AB}\\ -A_{BA} & A_{AA}\end{bmatrix}$
Losses and reactive power requirement of a two-port network	
Losses and reactive power requirement	$\underline{S}_V=P_V+jQ_V=\underline{S}_A+\underline{S}_B=\underline{U}_A\underline{I}_A^*+\underline{U}_B\underline{I}_B^*$

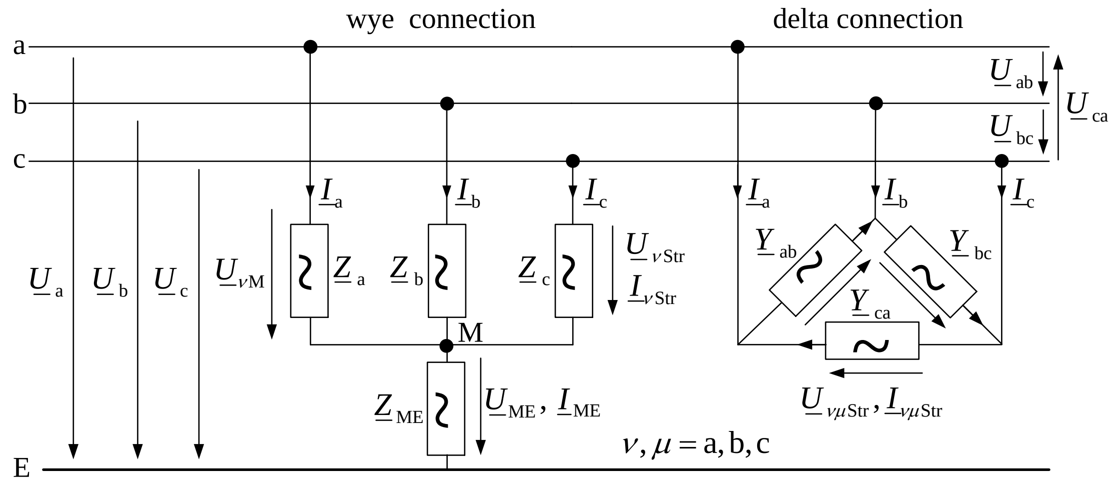
2.7.3 Special Two-Port Networks and Their Equivalent Circuits

Designation	Equivalent circuit	Iterative matrix $\underline{\boldsymbol{A}}$
Series impedance		$\begin{bmatrix} 1 & -\underline{Z} \\ 0 & -1 \end{bmatrix}$
Shunt admittance		$\begin{bmatrix} 1 & 0 \\ \underline{Y} & -1 \end{bmatrix}$
Ideal transformer		$\begin{bmatrix} \underline{ü} & 0 \\ 0 & -1/\underline{ü}^* \end{bmatrix}$
T-equivalent circuit		$\begin{bmatrix} \underline{Z}_1\underline{Y}_3 + 1 & -(\underline{Z}_1 + \underline{Z}_2 + \underline{Z}_1\underline{Z}_2\underline{Y}_3) \\ \underline{Y}_3 & -(\underline{Z}_2\underline{Y}_3 + 1) \end{bmatrix}$
Π-equivalent circuit		$\begin{bmatrix} \underline{Z}_3\underline{Y}_2 + 1 & -\underline{Z}_3 \\ \underline{Y}_1 + \underline{Y}_2 + \underline{Y}_1\underline{Y}_2\underline{Z}_3 & -(\underline{Z}_3\underline{Y}_1 + 1) \end{bmatrix}$
Uniform transmission line (see distributed parameters in 8.3)		$\begin{bmatrix} \cosh(\underline{\gamma}l) & -\underline{Z}_W \sinh(\underline{\gamma}l) \\ \underline{Y}_W \sinh(\underline{\gamma}l) & -\cosh(\underline{\gamma}l) \end{bmatrix}$

3 Three-Phase System

3.1 Wye Connection and Delta Connection

Three-phase system with two loads (wye connection and delta connection)



Line current	$\underline{I}_{\nu}$
Line-to-line voltage	$\underline{U}_{\nu\mu}$
Line-to-ground voltage	$\underline{U}_{\nu}$
Line-to-neutral voltage	$\underline{U}_{\nu M}$
Phase current	$\underline{I}_{\nu Str}$, $\underline{I}_{\nu\mu Str}$
Phase voltage	$\underline{U}_{\nu Str}$, $\underline{U}_{\nu\mu Str}$
Neutral-to-ground voltage	$\underline{U}_{ME}$
Neutral(-to-ground) current	$\underline{I}_{ME}$

Wye connection Y

Relation between phase, neutral and line currents and voltages:

$$\begin{bmatrix}\underline{U}_{ab}\\ \underline{U}_{bc}\\ \underline{U}_{ca}\end{bmatrix}=\begin{bmatrix}1 & -1 & 0\\ 0 & 1 & -1\\ -1 & 0 & 1\end{bmatrix}\begin{bmatrix}\underline{U}_{a}\\ \underline{U}_{b}\\ \underline{U}_{c}\end{bmatrix}=\begin{bmatrix}1 & -1 & 0\\ 0 & 1 & -1\\ -1 & 0 & 1\end{bmatrix}\begin{bmatrix}\underline{U}_{aStr}\\ \underline{U}_{bStr}\\ \underline{U}_{cStr}\end{bmatrix} \text{ and } \begin{bmatrix}\underline{I}_{a}\\ \underline{I}_{b}\\ \underline{I}_{c}\end{bmatrix}=\begin{bmatrix}\underline{I}_{aStr}\\ \underline{I}_{bStr}\\ \underline{I}_{cStr}\end{bmatrix}$$

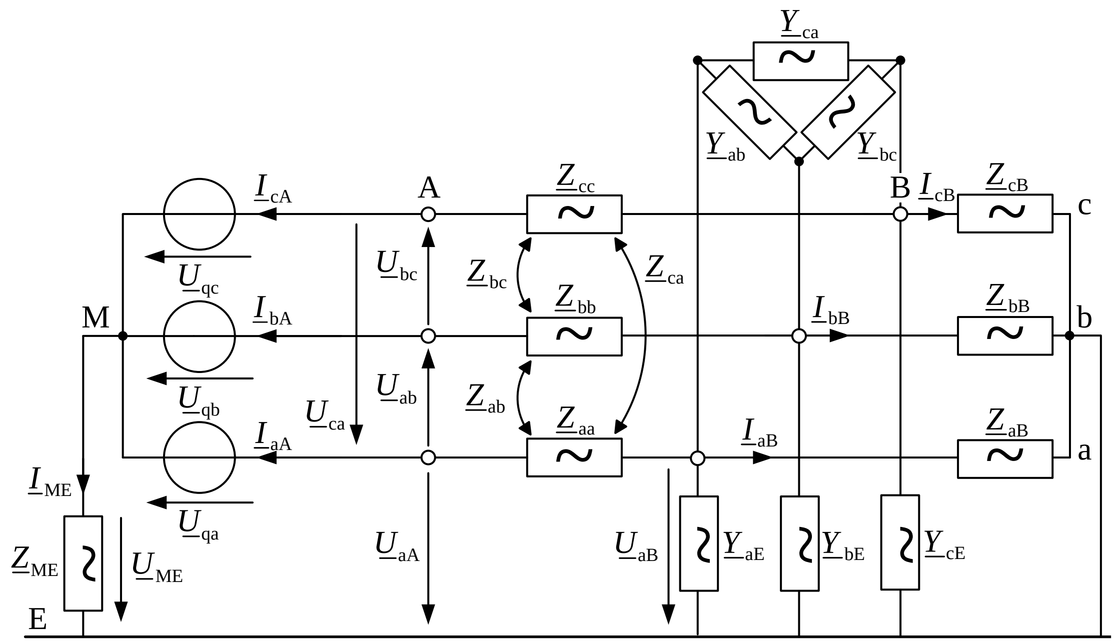
and $\underline{U}_{\nu} = \underline{U}_{\nu Str} + \underline{U}_{ME}$, $\underline{U}_{ME} = \underline{Z}_{ME}\underline{I}_{ME}$ and $\underline{I}_{ME} = \sum_{\nu}\underline{I}_{\nu} = \sum_{\nu}\underline{I}_{\nu Str}$ for $\nu = a, b, c$

Delta connection Δ

Relation between phase and line currents and voltages:

$$\begin{bmatrix}\underline{I}_{a}\\ \underline{I}_{b}\\ \underline{I}_{c}\end{bmatrix}=\begin{bmatrix}1 & 0 & -1\\ -1 & 1 & 0\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}\underline{I}_{abStr}\\ \underline{I}_{bcStr}\\ \underline{I}_{caStr}\end{bmatrix} \text{ and } \begin{bmatrix}\underline{U}_{ab}\\ \underline{U}_{bc}\\ \underline{U}_{ca}\end{bmatrix}=\begin{bmatrix}\underline{U}_{abStr}\\ \underline{U}_{bcStr}\\ \underline{U}_{caStr}\end{bmatrix}=\begin{bmatrix}1 & -1 & 0\\ 0 & 1 & -1\\ -1 & 0 & 1\end{bmatrix}\begin{bmatrix}\underline{U}_{a}\\ \underline{U}_{b}\\ \underline{U}_{c}\end{bmatrix}$$

3.2 Voltages and Currents in the Three-Phase System



Voltage equation system[2]:

$$\begin{bmatrix} \underline{Z}_{ME} & \underline{Z}_{ME} & \underline{Z}_{ME} \\ \underline{Z}_{ME} & \underline{Z}_{ME} & \underline{Z}_{ME} \\ \underline{Z}_{ME} & \underline{Z}_{ME} & \underline{Z}_{ME} \end{bmatrix} \begin{bmatrix} \underline{I}_{aA} \\ \underline{I}_{bA} \\ \underline{I}_{cA} \end{bmatrix} + \begin{bmatrix} \underline{U}_{qa} \\ \underline{U}_{qb} \\ \underline{U}_{qc} \end{bmatrix} + \begin{bmatrix} \underline{Z}_{aa} & \underline{Z}_{ab} & \underline{Z}_{ac} \\ \underline{Z}_{ba} & \underline{Z}_{bb} & \underline{Z}_{bc} \\ \underline{Z}_{ca} & \underline{Z}_{cb} & \underline{Z}_{cc} \end{bmatrix} \begin{bmatrix} \underline{I}_{aA} \\ \underline{I}_{bA} \\ \underline{I}_{cA} \end{bmatrix} = \begin{bmatrix} \underline{U}_{aB} \\ \underline{U}_{bB} \\ \underline{U}_{cB} \end{bmatrix}$$

$\underline{\boldsymbol{Z}}_{ME}\, \underline{\boldsymbol{i}}_{A} + \underline{\boldsymbol{u}}_{q} + \underline{\boldsymbol{Z}}\, \underline{\boldsymbol{i}}_{A} = \underline{\boldsymbol{u}}_{B}$ and

$$\begin{bmatrix} \underline{Z}_{aB} & 0 & 0 \\ 0 & \underline{Z}_{bB} & 0 \\ 0 & 0 & \underline{Z}_{cB} \end{bmatrix} \begin{bmatrix} \underline{I}_{aB} \\ \underline{I}_{bB} \\ \underline{I}_{cB} \end{bmatrix} = \begin{bmatrix} \underline{U}_{aB} \\ \underline{U}_{bB} \\ \underline{U}_{cB} \end{bmatrix}$$

$\underline{\boldsymbol{Z}}_{B}\, \underline{\boldsymbol{i}}_{B} = \underline{\boldsymbol{u}}_{B}$

including:

$$\begin{bmatrix} \underline{U}_{aA} \\ \underline{U}_{bA} \\ \underline{U}_{cA} \end{bmatrix} = \begin{bmatrix} \underline{U}_{qa} \\ \underline{U}_{qb} \\ \underline{U}_{qc} \end{bmatrix} + \begin{bmatrix} \underline{U}_{ME} \\ \underline{U}_{ME} \\ \underline{U}_{ME} \end{bmatrix} \quad \text{and} \quad \underline{U}_{ME} = \underline{Z}_{ME}\, \underline{I}_{ME} = \underline{Z}_{ME} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \underline{I}_{aA} \\ \underline{I}_{bA} \\ \underline{I}_{cA} \end{bmatrix}$$

$\underline{\boldsymbol{u}}_{A} = \underline{\boldsymbol{u}}_{q} + \underline{\boldsymbol{u}}_{ME}$ and $\underline{U}_{ME} = \underline{Z}_{ME}\, \mathbf{1}^{T}\, \underline{\boldsymbol{i}}_{A}$

Current equation system:

$$\begin{bmatrix} \underline{I}_{aA} \\ \underline{I}_{bA} \\ \underline{I}_{cA} \end{bmatrix} + \begin{bmatrix} \underline{I}_{aB} \\ \underline{I}_{bB} \\ \underline{I}_{cB} \end{bmatrix} + \begin{bmatrix} \underline{Y}_{aE} + \underline{Y}_{ab} + \underline{Y}_{ac} & -\underline{Y}_{ab} & -\underline{Y}_{ac} \\ -\underline{Y}_{ba} & \underline{Y}_{bE} + \underline{Y}_{ba} + \underline{Y}_{bc} & -\underline{Y}_{bc} \\ -\underline{Y}_{ca} & -\underline{Y}_{cb} & \underline{Y}_{cE} + \underline{Y}_{ca} + \underline{Y}_{cb} \end{bmatrix} \begin{bmatrix} \underline{U}_{aB} \\ \underline{U}_{bB} \\ \underline{U}_{cB} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\underline{\boldsymbol{i}}_{A} + \underline{\boldsymbol{i}}_{B} + \underline{\boldsymbol{Y}}\, \underline{\boldsymbol{u}}_{B} = 0$

---

[2] The connection between the phases due to the mutual impedances is neglected for $\underline{\boldsymbol{Z}}_{B}$. Taking into account the mutual impedances requires $\underline{\boldsymbol{Z}}_{B}$ to be build up in analogy to $\underline{\boldsymbol{Z}}$.

3.3 Balanced Three-Phase System

3.3.1 Conditions for a balanced three-phase system

a) Geometric symmetry (symmetrical structure of the network components)

Impedance matrix (see Section 3.2):

$$\underline{\boldsymbol{Z}} = \begin{bmatrix} \underline{Z}_{aa} & \underline{Z}_{ab} & \underline{Z}_{ac} \\ \underline{Z}_{ba} & \underline{Z}_{bb} & \underline{Z}_{bc} \\ \underline{Z}_{ca} & \underline{Z}_{cb} & \underline{Z}_{cc} \end{bmatrix} = \begin{bmatrix} \underline{Z}_{s} & \underline{Z}_{g} & \underline{Z}_{g} \\ \underline{Z}_{g} & \underline{Z}_{s} & \underline{Z}_{g} \\ \underline{Z}_{g} & \underline{Z}_{g} & \underline{Z}_{s} \end{bmatrix}$$

including $\underline{Z}_{aa} = \underline{Z}_{bb} = \underline{Z}_{cc} = \underline{Z}_{s}$ and $\underline{Z}_{ab} = \underline{Z}_{ba} = \underline{Z}_{bc} = \underline{Z}_{cb} = \underline{Z}_{ac} = \underline{Z}_{ca} = \underline{Z}_{g}$

Admittance matrix (see Section 3.2):

$$\underline{\boldsymbol{Y}} = \begin{bmatrix} \underline{Y}_{aE} + \underline{Y}_{ab} + \underline{Y}_{ac} & -\underline{Y}_{ab} & -\underline{Y}_{ac} \\ -\underline{Y}_{ba} & \underline{Y}_{bE} + \underline{Y}_{ba} + \underline{Y}_{bc} & -\underline{Y}_{bc} \\ -\underline{Y}_{ca} & -\underline{Y}_{cb} & \underline{Y}_{cE} + \underline{Y}_{ca} + \underline{Y}_{cb} \end{bmatrix} = \begin{bmatrix} \underline{Y}_{s} & \underline{Y}_{g} & \underline{Y}_{g} \\ \underline{Y}_{g} & \underline{Y}_{s} & \underline{Y}_{g} \\ \underline{Y}_{g} & \underline{Y}_{g} & \underline{Y}_{s} \end{bmatrix}$$ including

$\underline{Y}_{aE} = \underline{Y}_{bE} = \underline{Y}_{cE} = \underline{Y}_{E}$, $\underline{Y}_{ab} = \underline{Y}_{ba} = \underline{Y}_{bc} = \underline{Y}_{cb} = \underline{Y}_{ac} = \underline{Y}_{ca} = \underline{Y} = -\underline{Y}_{g}$ and $\underline{Y}_{E} + 2\underline{Y} = \underline{Y}_{s}$

b) Electric symmetry (symmetrical (balanced) sources and loads)

Symmetrical sources	$\underline{U}_{qa} + \underline{U}_{qb} + \underline{U}_{qc} = \underline{U}_{qa} + \underline{a}^2\underline{U}_{qa} + \underline{a}\underline{U}_{qa} = 0$
Symmetrical loads	$\underline{Z}_{aB} = \underline{Z}_{bB} = \underline{Z}_{cB} = \underline{Z}_{B}$ (see footnote 1)

From a) and b) follows: Symmetrical currents and voltages at each location *x*

$\underline{I}_{ax} + \underline{I}_{bx} + \underline{I}_{cx} = \underline{I}_{ax}(1 + \underline{a}^2 + \underline{a}) = 0$ and $\underline{U}_{ax} + \underline{U}_{bx} + \underline{U}_{cx} = \underline{U}_{ax}(1 + \underline{a}^2 + \underline{a}) = 0$

3.3.2 Single-Phase Equivalent Circuit

Assuming a balanced three-phase system as described in 3.3.1, the general voltage and current equation systems from 3.2 can be transferred to decoupled equation systems:

$$\begin{bmatrix} \underline{U}_{qa} \\ \underline{U}_{qb} \\ \underline{U}_{qc} \end{bmatrix} + \begin{bmatrix} \underline{Z}_{s} - \underline{Z}_{g} & 0 & 0 \\ 0 & \underline{Z}_{s} - \underline{Z}_{g} & 0 \\ 0 & 0 & \underline{Z}_{s} - \underline{Z}_{g} \end{bmatrix} \begin{bmatrix} \underline{I}_{aA} \\ \underline{I}_{bA} \\ \underline{I}_{cA} \end{bmatrix} = \begin{bmatrix} \underline{U}_{aB} \\ \underline{U}_{bB} \\ \underline{U}_{cB} \end{bmatrix} = \begin{bmatrix} \underline{Z}_{B} & 0 & 0 \\ 0 & \underline{Z}_{B} & 0 \\ 0 & 0 & \underline{Z}_{B} \end{bmatrix} \begin{bmatrix} \underline{I}_{aB} \\ \underline{I}_{bB} \\ \underline{I}_{cB} \end{bmatrix}$$ and

$$\begin{bmatrix} \underline{I}_{aA} \\ \underline{I}_{bA} \\ \underline{I}_{cA} \end{bmatrix} + \begin{bmatrix} \underline{I}_{aB} \\ \underline{I}_{bB} \\ \underline{I}_{cB} \end{bmatrix} + \begin{bmatrix} \underline{Y}_{s} - \underline{Y}_{g} & 0 & 0 \\ 0 & \underline{Y}_{s} - \underline{Y}_{g} & 0 \\ 0 & 0 & \underline{Y}_{s} - \underline{Y}_{g} \end{bmatrix} \begin{bmatrix} \underline{U}_{aB} \\ \underline{U}_{bB} \\ \underline{U}_{cB} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ with $\underline{U}_{ME} = 0$ and $\underline{I}_{ME} = 0$ as well as $\underline{\boldsymbol{u}}_{A} = \underline{\boldsymbol{u}}_{q}$

Single-phase equivalent circuit for the reference phase a of the balanced three-phase system from Section 3.2

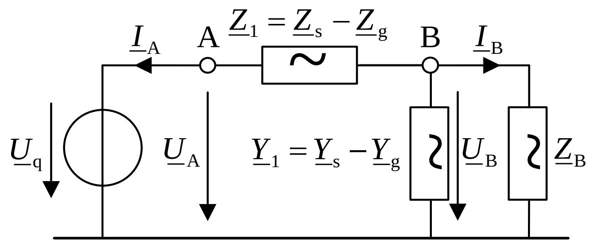
With:

$\underline{Z}_{1} = \underline{Z}_{s} - \underline{Z}_{g}$,

$\underline{Y}_{1} = \underline{Y}_{s} - \underline{Y}_{g} = \underline{Y}_{E} + 3\underline{Y}$,

and $\underline{U}_{q} = \underline{U}_{qa}$

(The index a can be omitted)



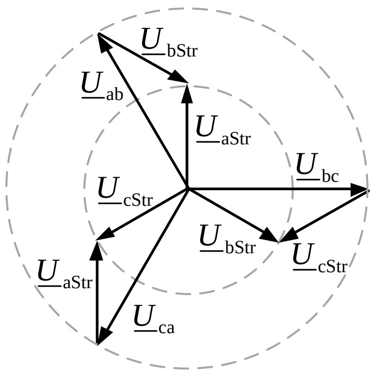
The single-phase equivalent circuit for the line a of the balanced three-phase system is identical to the positive-sequence equivalent circuit (index 1) (see Section 3.5.2).

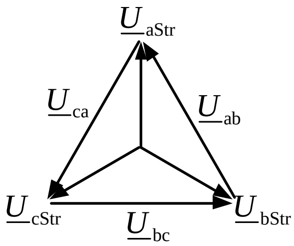
3.3.3 Relations Between Phase, Neutral and Line Voltages and Currents

Wye connection Y

Relation between phase, neutral and line currents and voltages:

$$\begin{bmatrix}\underline{U}_{\mathrm{ab}}\\ \underline{U}_{\mathrm{bc}}\\ \underline{U}_{\mathrm{ca}}\end{bmatrix} = \left(1-\underline{\mathrm{a}}^2\right)\begin{bmatrix}\underline{U}_{\mathrm{aStr}}\\ \underline{U}_{\mathrm{bStr}}\\ \underline{U}_{\mathrm{cStr}}\end{bmatrix} = \sqrt{3}\,\mathrm{e}^{\mathrm{j}\frac{\pi}{6}}\begin{bmatrix}\underline{U}_{\mathrm{aStr}}\\ \underline{U}_{\mathrm{bStr}}\\ \underline{U}_{\mathrm{cStr}}\end{bmatrix} \quad \text{and} \quad \begin{bmatrix}\underline{I}_{\mathrm{a}}\\ \underline{I}_{\mathrm{b}}\\ \underline{I}_{\mathrm{c}}\end{bmatrix} = \begin{bmatrix}\underline{I}_{\mathrm{aStr}}\\ \underline{I}_{\mathrm{bStr}}\\ \underline{I}_{\mathrm{cStr}}\end{bmatrix} \quad \text{and} \quad \underline{U}_{\nu\mathrm{Str}} = \underline{U}_{\nu} = \underline{U}_{\nu\mathrm{M}}$$





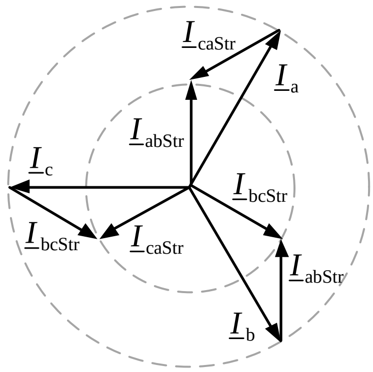
$\underline{I}_{\nu} = \underline{I}_{\nu\mathrm{Str}}$

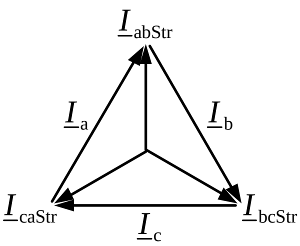
$\nu = \mathrm{a, b, c}$

Delta connection Δ

Relation between phase and line currents and voltages:

$$\begin{bmatrix}\underline{I}_{\mathrm{a}}\\ \underline{I}_{\mathrm{b}}\\ \underline{I}_{\mathrm{c}}\end{bmatrix} = \left(1-\underline{\mathrm{a}}\right)\begin{bmatrix}\underline{I}_{\mathrm{abStr}}\\ \underline{I}_{\mathrm{bcStr}}\\ \underline{I}_{\mathrm{caStr}}\end{bmatrix} = \sqrt{3}\,\mathrm{e}^{-\mathrm{j}\frac{\pi}{6}}\begin{bmatrix}\underline{I}_{\mathrm{abStr}}\\ \underline{I}_{\mathrm{bcStr}}\\ \underline{I}_{\mathrm{caStr}}\end{bmatrix} \quad \text{and} \quad \begin{bmatrix}\underline{U}_{\mathrm{ab}}\\ \underline{U}_{\mathrm{bc}}\\ \underline{U}_{\mathrm{ca}}\end{bmatrix} = \begin{bmatrix}\underline{U}_{\mathrm{abStr}}\\ \underline{U}_{\mathrm{bcStr}}\\ \underline{U}_{\mathrm{caStr}}\end{bmatrix} = \sqrt{3}\,\mathrm{e}^{\mathrm{j}\frac{\pi}{6}}\begin{bmatrix}\underline{U}_{\mathrm{a}}\\ \underline{U}_{\mathrm{b}}\\ \underline{U}_{\mathrm{c}}\end{bmatrix}$$

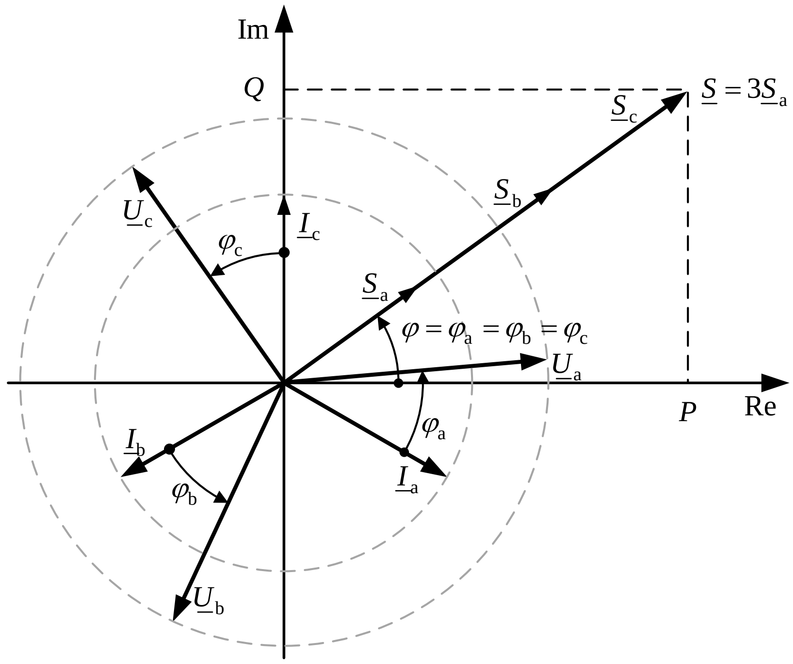




$\underline{U}_{\nu\mu\mathrm{Str}} = \underline{U}_{\nu\mu}$

$\nu, \mu = \mathrm{a, b, c}$

3.3.4 Phasor Diagram and Three-Phase Apparent Power



Three-phase apparent power	$\underline{S} = \underline{S}_a + \underline{S}_b + \underline{S}_c = \underline{U}_a \underline{I}_a^* + \underline{U}_b \underline{I}_b^* + \underline{U}_c \underline{I}_c^*$ $= 3U_a I_a \, e^{j\varphi} = P + j\,Q = S\cos(\varphi) + j\,S\sin(\varphi)$ $= S\,e^{j\varphi}$

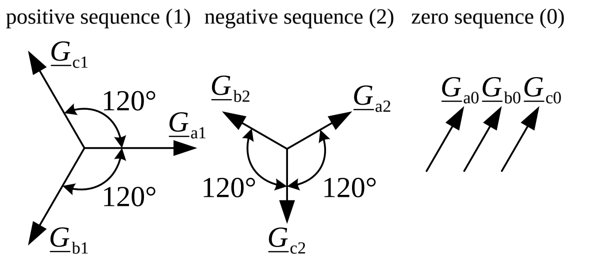
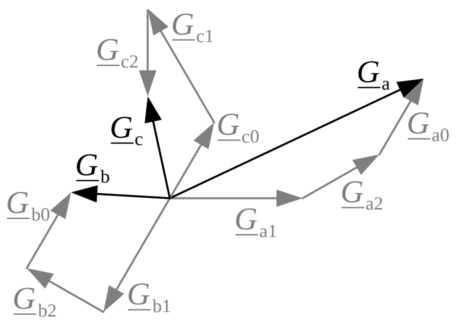
3.4 Wye-Delta Transformation

Transformations between delta and wye connections[3)]	
Wye-delta transformation (Y→Δ)	$\underline{Y}_{ab} = \frac{\underline{Y}_a \underline{Y}_b}{\underline{Y}_\Sigma}$, $\underline{Y}_{bc} = \frac{\underline{Y}_b \underline{Y}_c}{\underline{Y}_\Sigma}$, $\underline{Y}_{ca} = \frac{\underline{Y}_c \underline{Y}_a}{\underline{Y}_\Sigma}$ including $\underline{Y}_\Sigma = \underline{Y}_a + \underline{Y}_b + \underline{Y}_c$
Delta-wye transformation (Δ→Y)	$\underline{Z}_a = \frac{\underline{Z}_{ab} \underline{Z}_{ca}}{\underline{Z}_\Sigma}$, $\underline{Z}_b = \frac{\underline{Z}_{ab} \underline{Z}_{bc}}{\underline{Z}_\Sigma}$, $\underline{Z}_c = \frac{\underline{Z}_{ac} \underline{Z}_{bc}}{\underline{Z}_\Sigma}$ including $\underline{Z}_\Sigma = \underline{Z}_{ab} + \underline{Z}_{bc} + \underline{Z}_{ca}$
Transformation for the balanced three-phase system (see Section 3.3)	
$\underline{Y}_\nu = \underline{Y}_Y$ resp. $\underline{Z}_{\nu\mu} = \underline{Z}_\Delta$ with $\nu, \mu = a, b, c$	$\underline{Y}_{\nu\mu} = \frac{\underline{Y}_Y}{3}$ for Y→Δ and $\underline{Z}_\nu = \frac{\underline{Z}_\Delta}{3}$ for Δ→Y

3) Requirement: No grounding of the neutral point of the wye circuit ($\underline{Y}_{ME} = 0$ resp. $|\underline{Z}_{ME}| \to \infty$).

3.5 Unbalanced Three-Phase System

3.5.1 Symmetrical Components

Symmetrical components (SC) of an unbalanced three-phase system	
Phase a as reference phase	$\underline{G}_{a1}=\underline{G}_1 \quad \underline{G}_{a2}=\underline{G}_2 \quad \underline{G}_{a0}=\underline{G}_0$ $\underline{G}_{b1}=\underline{a}^2\underline{G}_1 \quad \underline{G}_{b2}=\underline{a}\,\underline{G}_2 \quad \underline{G}_{b0}=\underline{G}_0$ $\underline{G}_{c1}=\underline{a}\,\underline{G}_1 \quad \underline{G}_{c2}=\underline{a}^2\underline{G}_2 \quad \underline{G}_{c0}=\underline{G}_0$
Decomposition of quantities into symmetrical components	$\underline{\boldsymbol{g}} = \begin{bmatrix}\underline{G}_a\\ \underline{G}_b\\ \underline{G}_c\end{bmatrix} = \begin{bmatrix}\underline{G}_{a1}+\underline{G}_{a2}+\underline{G}_{a0}\\ \underline{G}_{b1}+\underline{G}_{b2}+\underline{G}_{b0}\\ \underline{G}_{c1}+\underline{G}_{c2}+\underline{G}_{c0}\end{bmatrix}$
Transformation from phase quantities a, b, c to symmetrical components 1, 2, 0 and its inverse transformation	$\underline{\boldsymbol{g}} = \begin{bmatrix}\underline{G}_a\\ \underline{G}_b\\ \underline{G}_c\end{bmatrix} = \begin{bmatrix}1 & 1 & 1\\ \underline{a}^2 & \underline{a} & 1\\ \underline{a} & \underline{a}^2 & 1\end{bmatrix}\begin{bmatrix}\underline{G}_1\\ \underline{G}_2\\ \underline{G}_0\end{bmatrix} = \underline{\boldsymbol{T}}_S\underline{\boldsymbol{g}}_S$ $\underline{\boldsymbol{g}}_S = \begin{bmatrix}\underline{G}_1\\ \underline{G}_2\\ \underline{G}_0\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & \underline{a} & \underline{a}^2\\ 1 & \underline{a}^2 & \underline{a}\\ 1 & 1 & 1\end{bmatrix}\begin{bmatrix}\underline{G}_a\\ \underline{G}_b\\ \underline{G}_c\end{bmatrix} = \underline{\boldsymbol{T}}_S^{-1}\underline{\boldsymbol{g}}$
Phasor diagram of unbalanced phase quantities and their composition in symmetrical components	
Apparent power in symmetrical components	$\underline{S}_S = \underline{\boldsymbol{u}}_S^T\underline{\boldsymbol{i}}_S^* = \underline{U}_1\underline{I}_1^* + \underline{U}_2\underline{I}_2^* + \underline{U}_0\underline{I}_0^*$
Apparent power in phase quantities	$\underline{S} = \underline{\boldsymbol{u}}^T\underline{\boldsymbol{i}}^* = \underline{U}_a\underline{I}_a^* + \underline{U}_b\underline{I}_b^* + \underline{U}_c\underline{I}_c^*$
Transformation to symmetrical components is variant in terms of apparent power	$\underline{S} = \underline{\boldsymbol{u}}^T\underline{\boldsymbol{i}}^* = (\underline{\boldsymbol{T}}_S\underline{\boldsymbol{u}}_S)^T(\underline{\boldsymbol{T}}_S\underline{\boldsymbol{i}}_S)^* = \underline{\boldsymbol{u}}_S^T\underline{\boldsymbol{T}}_S^T\underline{\boldsymbol{T}}_S^*\underline{\boldsymbol{i}}_S^*$ $= 3\underline{\boldsymbol{u}}_S^T\underline{\boldsymbol{i}}_S^* = 3\,\underline{S}_S$

3.5.2 Equivalent Circuits in Symmetrical Components

Voltage equations from 3.2 in symmetrical components (decoupled):

$$\begin{bmatrix} 0 & & \\ & 0 & \\ & & 3\underline{Z}_{ME} \end{bmatrix}\begin{bmatrix} \underline{I}_{1A} \\ \underline{I}_{2A} \\ \underline{I}_{0A} \end{bmatrix} + \begin{bmatrix} \underline{U}_{q1} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \underline{Z}_1 & & \\ & \underline{Z}_2 & \\ & & \underline{Z}_0 \end{bmatrix}\begin{bmatrix} \underline{I}_{1A} \\ \underline{I}_{2A} \\ \underline{I}_{0A} \end{bmatrix} = \begin{bmatrix} \underline{U}_{1B} \\ \underline{U}_{2B} \\ \underline{U}_{0B} \end{bmatrix} = \begin{bmatrix} \underline{Z}_{1B} & 0 & 0 \\ 0 & \underline{Z}_{2B} & 0 \\ 0 & 0 & \underline{Z}_{0B} \end{bmatrix}\begin{bmatrix} \underline{I}_{1B} \\ \underline{I}_{2B} \\ \underline{I}_{0B} \end{bmatrix}$$

Current equations from 3.2 in symmetrical components (decoupled):

$$\begin{bmatrix} \underline{I}_{1A} \\ \underline{I}_{2A} \\ \underline{I}_{0A} \end{bmatrix} + \begin{bmatrix} \underline{I}_{1B} \\ \underline{I}_{2B} \\ \underline{I}_{0B} \end{bmatrix} + \begin{bmatrix} \underline{Y}_1 & & \\ & \underline{Y}_2 & \\ & & \underline{Y}_0 \end{bmatrix}\begin{bmatrix} \underline{U}_{1B} \\ \underline{U}_{2B} \\ \underline{U}_{0B} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{with} \quad \begin{bmatrix} \underline{U}_{1A} \\ \underline{U}_{2A} \\ \underline{U}_{0A} \end{bmatrix} = \begin{bmatrix} \underline{U}_{q1} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 3\underline{Z}_{ME}\underline{I}_{0A} \end{bmatrix}$$

Positive-sequence equivalent circuit:

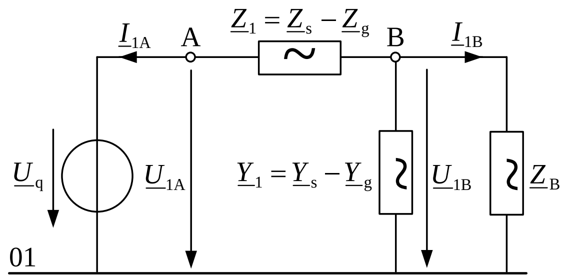
$\underline{Z}_1 = \underline{Z}_s - \underline{Z}_g$

$\underline{Z}_{1B} = \underline{Z}_B$

$\underline{Y}_1 = \underline{Y}_s - \underline{Y}_g = \underline{Y}_E + 3\underline{Y}$

$\underline{U}_{q1} = \underline{U}_{qa} = \underline{U}_q$

(compare to single-phase equivalent circuit of the symmetrical 3-phase system in 3.3.1)

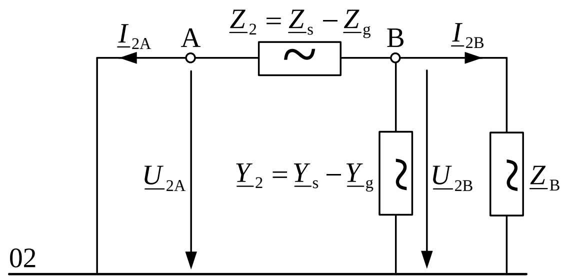


Negative-sequence equivalent circuit:

$\underline{Z}_2 = \underline{Z}_1 = \underline{Z}_s - \underline{Z}_g$

$\underline{Z}_{2B} = \underline{Z}_{1B} = \underline{Z}_B$

$\underline{Y}_2 = \underline{Y}_1 = \underline{Y}_s - \underline{Y}_g = \underline{Y}_E + 3\underline{Y}$

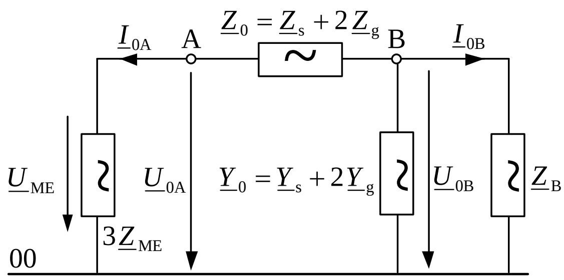


Zero-sequence equivalent circuit:

$\underline{Z}_0 = \underline{Z}_s + 2\underline{Z}_g$

$\underline{Z}_{0B} = \underline{Z}_B$

$\underline{Y}_0 = \underline{Y}_s + 2\underline{Y}_g = \underline{Y}_E$



The neutral-to-ground impedance $\underline{Z}_{ME}$ affects only the zero-sequence system. The neutral-to-ground voltage is: $\underline{U}_{ME} = 3\underline{Z}_{ME}\underline{I}_{0A}$

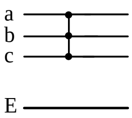
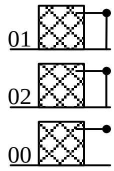
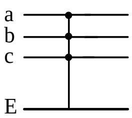
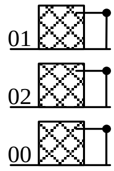
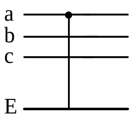
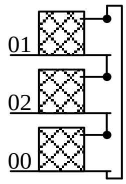
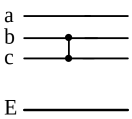
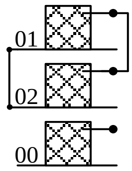
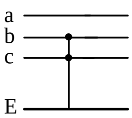
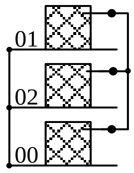
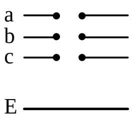
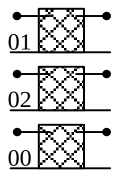
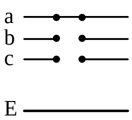
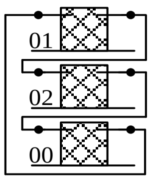
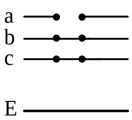
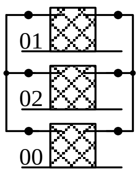
For the balanced three-phase system, the following applies:

The component systems are decoupled and the negative- and zero-sequence equivalent circuits are passive networks. Thus, the balanced three-system can be described by the positive-sequence equivalent circuit (see 3.3.1).

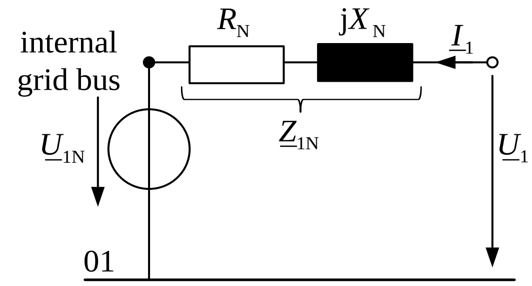
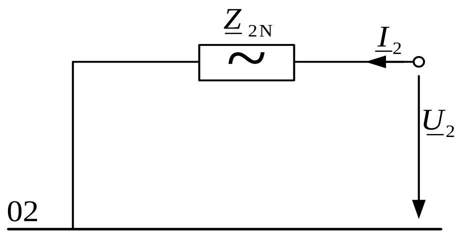
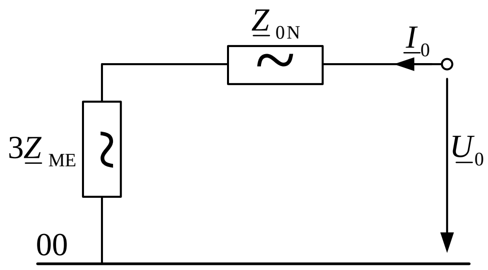
For the unbalanced three-phase system, the following applies:

As a result of unbalanced faults, the equivalent circuits of the symmetrical components are coupled at the fault location (see 3.5.3) The structure of coupling follows from three fault conditions, which have to be transferred to symmetrical components.

3.5.3 Fault Conditions and Interconnections of Equivalent Circuits in Symmetrical Components

Fault type	Fault conditions				Interconnection of the SC
	Three-phase system		Symmetrical components (SC)		
	$\underline{U}_a - \underline{U}_b = 0$ $\underline{U}_b - \underline{U}_c = 0$ —	— — $\underline{I}_a + \underline{I}_b + \underline{I}_c = 0$	$\underline{U}_1 = 0$ $\underline{U}_2 = 0$ —	— — $\underline{I}_0 = 0$	
	$\underline{U}_a = 0$ $\underline{U}_b = 0$ $\underline{U}_c = 0$	— — —	$\underline{U}_1 = 0$ $\underline{U}_2 = 0$ $\underline{U}_0 = 0$	— — —	
	$\underline{U}_a = 0$ — —	— $\underline{I}_b = 0$ $\underline{I}_c = 0$	$\underline{U}_1 + \underline{U}_2 + \underline{U}_0 = 0$ — —	— $\underline{I}_2 = \underline{I}_1$ $\underline{I}_0 = \underline{I}_2$	
	— — $\underline{U}_b - \underline{U}_c = 0$	$\underline{I}_a = 0$ $\underline{I}_b + \underline{I}_c = 0$ —	— — $\underline{U}_2 = \underline{U}_1$	$\underline{I}_1 + \underline{I}_2 = 0$ $\underline{I}_0 = 0$ —	
	— $\underline{U}_b = 0$ $\underline{U}_c = 0$	$\underline{I}_a = 0$ — —	— $\underline{U}_2 = \underline{U}_1$ $\underline{U}_0 = \underline{U}_2$	$\underline{I}_1 + \underline{I}_2 + \underline{I}_0 = 0$ — —	
	— — —	$\underline{I}_a = 0$ $\underline{I}_b = 0$ $\underline{I}_c = 0$	— — —	$\underline{I}_1 = 0$ $\underline{I}_2 = 0$ $\underline{I}_0 = 0$	
	$\underline{U}_a = 0$ — —	— $\underline{I}_b = 0$ $\underline{I}_c = 0$	$\underline{U}_1 + \underline{U}_2 + \underline{U}_0 = 0$ — —	— $\underline{I}_2 = \underline{I}_1$ $\underline{I}_0 = \underline{I}_2$	
	— $\underline{U}_b = 0$ $\underline{U}_c = 0$	$\underline{I}_a = 0$ — —	— $\underline{U}_2 = \underline{U}_1$ $\underline{U}_0 = \underline{U}_2$	$\underline{I}_1 + \underline{I}_2 + \underline{I}_0 = 0$ — —	

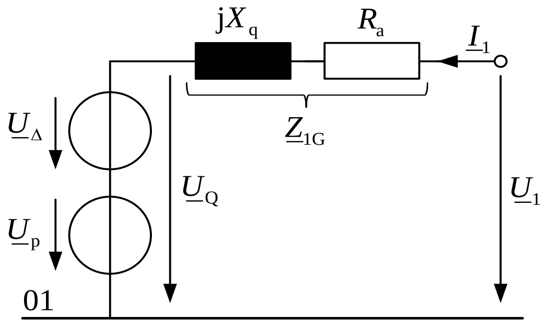
4 Equivalent Networks

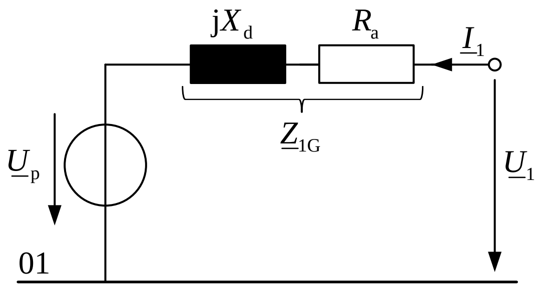
Positive-sequence equivalent circuit	
Internal grid impedance: $\underline{Z}_1 = \underline{Z}_{1N} = R_N + jX_N$	
Positive-sequence impedance (see Section 12.3 for voltage factor $c = c_{max} = 1{,}1$)	$Z_{1N} = \frac{cU_{nN}}{\sqrt{3}\,I''_k} = \frac{cU^2_{nN}}{S''_k} = X_{1N}\sqrt{1+\left(\frac{r_N}{x_N}\right)^2}$
R-*X* ratio:	$r_N / x_N = \frac{R_N}{X_N}$
Three-phase short-circuit power (fictitious calculation value, see Section 12.3 for voltage factor $c = c_{max} = 1{,}1$)	$S''_k = \sqrt{3}U_{nN}\,I''_k = \sqrt{3}U_{nN}\frac{cU_{nN}}{\sqrt{3}\,Z_{1N}} = \frac{cU^2_{nN}}{Z_{1N}}$
Negative-sequence equivalent circuit	
Usually, the negative-sequence impedance for equivalent circuits is: $\underline{Z}_2 = \underline{Z}_{2N} = \underline{Z}_{1N}$	
Zero-sequence equivalent circuit	
The zero-sequence impedance $\underline{Z}_0 = \underline{Z}_{0N} + 3\underline{Z}_{ME}$ depends on the neutral grounding: $\frac{Z_0}{Z_{1N}} = 3\frac{I''_k}{I''_{k1}} - 2$ (I''_k / I''_{k1} three-phase to single-phase short-circuit ratio)	
For isolated-neutral and resonant-grounded power systems, the following applies: $Z_0 \gg Z_{1N}$ For solidly grounded ($Z_{ME} = 0$) power systems, the following applies: $Z_0 = Z_{0N} > Z_{1N}$	
Special case: stiff network	
$S''_k \to \infty$ and $\underline{Z}_{1N} = \underline{Z}_{2N} = \underline{Z}_{0N} = \underline{Z}_{ME} \to 0$	

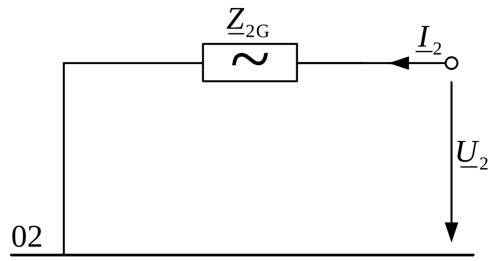
5 Synchronous Machines

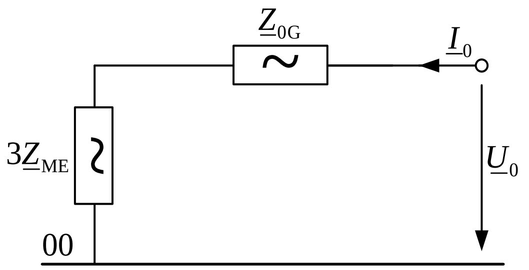
5.1 Equivalent Circuits for Steady-State Operating Conditions

<table>
<tr><th colspan="2">Positive-sequence equivalent circuit</th></tr>
<tr><td>General generator equation</td><td>$\underline{U}_1 = \left(R_a + \mathrm{j}X_1\right)\underline{I}_1 + \underline{U}_\Delta + \underline{U}_p$
$\underline{U}_\Delta = \mathrm{j}\left(X_d - X_1\right)\underline{I}_{1d} + \mathrm{j}\left(X_q - X_1\right)\underline{I}_{1q}$</td></tr>
<tr><td colspan="2">Positive-sequence equivalent circuit of salient-pole machines</td></tr>
<tr><td>Salient-pole machine: $X_d \neq X_q$
Appropriate: $X_1 = X_q$

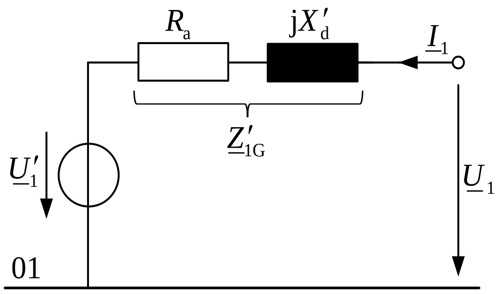
Positive-sequence impedance:
$\underline{Z}_1 = \underline{Z}_{1G} = R_a + \mathrm{j}X_1$</td><td>
</td></tr>
<tr><td colspan="2">Positive-sequence equivalent circuit of nonsalient-pole machines (or cylindrical-rotor machines)</td></tr>
<tr><td>Nonsalient-pole machine: $X_d = X_q$
Appropriate: $X_1 = X_d$

Positive-sequence impedance:
$\underline{Z}_1 = \underline{Z}_{1G} = R_a + \mathrm{j}X_1$</td><td>
</td></tr>
<tr><td>Synchronous generated voltage (nonlinear function of the excitation current, no-load characteristic) and field ratio</td><td>$U_p = f\left(I_f'\right)$ and $\varepsilon = \dfrac{U_p}{U_{rG}/\sqrt{3}}$</td></tr>
<tr><td>No-load excitation current I_{f0}'</td><td>$U_p\left(I_{f0}'\right) = U_{rG}/\sqrt{3}$ and $\varepsilon = 1$</td></tr>
<tr><td>Over- or underexcitation</td><td>$I_f' > I_{f0}'$ and $\varepsilon > 1$ resp. $I_f' < I_{f0}'$ and $\varepsilon < 1$</td></tr>
<tr><td>Rotor displacement angle</td><td>$\delta_p = \measuredangle\left(\underline{U}_p, \underline{U}_1\right) = \varphi_{Up} - \varphi_{U1}$</td></tr>
<tr><td>Reference impedance</td><td>$Z_B = \dfrac{U_{rG}}{\sqrt{3}I_{rG}} = \dfrac{U_{rG}^2}{S_{rG}}$</td></tr>
<tr><td>Armature resistance</td><td>$R_a = r_a\, Z_B$</td></tr>
<tr><td>Direct-axis synchronous reactance</td><td>$X_d = x_d\, Z_B$</td></tr>
<tr><td>Quadrature-axis synchronous reactance</td><td>$X_q = x_q\, Z_B$</td></tr>
</table>

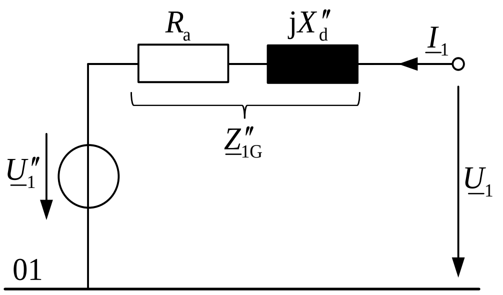
Negative-sequence equivalent circuit	
Negative-sequence impedance: $\underline{Z}_2 = \underline{Z}_{2G} = R_a + \frac{j}{2}(X''_d + X''_q) + \frac{1}{4}(k_f^2 R_f + k_D^2 R_D + k_Q^2 R_Q)$	

Zero-sequence equivalent circuit	
Zero-sequence impedance: $\underline{Z}_0 = \underline{Z}_{0G} + 3\underline{Z}_{ME} = R_a + jX_{0G} + 3R_{ME} + 3jX_{ME}$	
Typically, the neutral points of synchronous machines are not grounded: $Z_{ME} \rightarrow \infty$	

5.2 Equivalent Circuit for Transient Operating Conditions

Positive-sequence equivalent circuit of nonsalient-pole machines	
Direct-axis transient reactance: $X'_d = x'_d \, Z_B$ Transient voltage: $\underline{U}'_1 = \underline{U}_1(0^-) - (R_a + jX'_d)\,\underline{I}_1(0^-) = \underline{U}_1(0^-) - \underline{Z}'_{1G}\,\underline{I}_1(0^-)$	
The transient voltage $\underline{U}'_1$ (and the subtransient voltage $\underline{U}''_1$ in Section 5.3) are determined using the terminal voltage and current immediately before ($t = 0^-$) the fault	

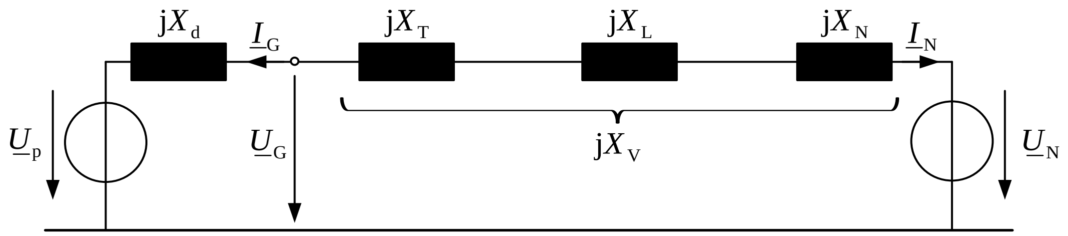
5.3 Equivalent Circuit for Subtransient Operating Conditions

Positive-sequence equivalent circuit of nonsalient-pole machines	
Direct-axis subtransient reactance: $X''_d = x''_d \, Z_B$ Subtransient voltage: $\underline{U}''_1 = \underline{U}_1(0^-) - (R_a + jX''_d)\,\underline{I}_1(0^-) = \underline{U}_1(0^-) - \underline{Z}''_{1G}\,\underline{I}_1(0^-)$	

5.4 Synchronous Grid Operation

Example: Synchronous machine supplies power to a grid via a transformer and a line

Positive-sequence equivalent circuit (converted to the same voltage level):



with the synchronous generated voltage $\underline{U}_\mathrm{p} = U_\mathrm{p}\,\mathrm{e}^{\mathrm{j}\varphi_\mathrm{Up}}$ and the internal grid voltage[4)] $\underline{U}_\mathrm{N} = U_\mathrm{N}\,\mathrm{e}^{\mathrm{j}\varphi_\mathrm{UN}}$

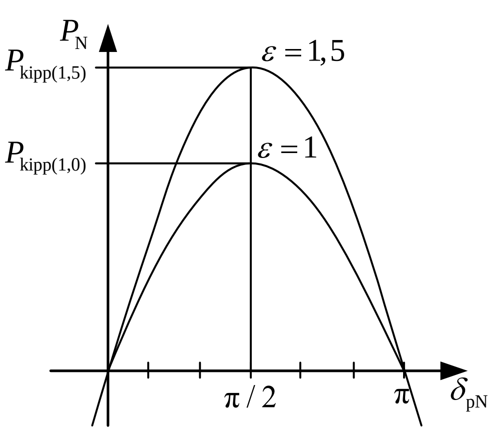
Apparent power supplied to the internal bus of the grid[5)]	$\underline{S}_\mathrm{N} = P_\mathrm{N} + \mathrm{j}Q_\mathrm{N} = 3\underline{U}_\mathrm{N}\underline{I}_\mathrm{N}^{*}$ $= \mathrm{j}\dfrac{3U_\mathrm{p}U_\mathrm{N}}{X_\mathrm{d}+X_\mathrm{V}}\mathrm{e}^{-\mathrm{j}\delta_\mathrm{pN}} - \mathrm{j}\dfrac{3U_\mathrm{N}^2}{X_\mathrm{d}+X_\mathrm{V}}$ $= \mathrm{j}\varepsilon Q_\mathrm{C}\,\mathrm{e}^{-\mathrm{j}\delta_\mathrm{pN}} - \mathrm{j}Q_\mathrm{C}$
Maximum active power supplied to the internal bus of the grid	$P_\mathrm{kipp} = 3\dfrac{U_\mathrm{p}U_\mathrm{N}}{X_\mathrm{d}+X_\mathrm{V}}$
Maximum reactive power supplied at the internal bus of the grid for $\varepsilon = 0$	$Q_\mathrm{C} = 3\dfrac{U_\mathrm{N}^2}{X_\mathrm{d}+X_\mathrm{V}}$
Resulting rotor displacement angle[4)] (Reference: internal grid voltage)	$\delta_\mathrm{pN} = \measuredangle\left(\underline{U}_\mathrm{p}, \underline{U}_\mathrm{N}\right) = \varphi_\mathrm{Up} - \varphi_\mathrm{UN}$
Resulting field ratio[4)] (Reference: internal grid voltage)	$\varepsilon = \dfrac{U_\mathrm{p}}{U_\mathrm{N}}$

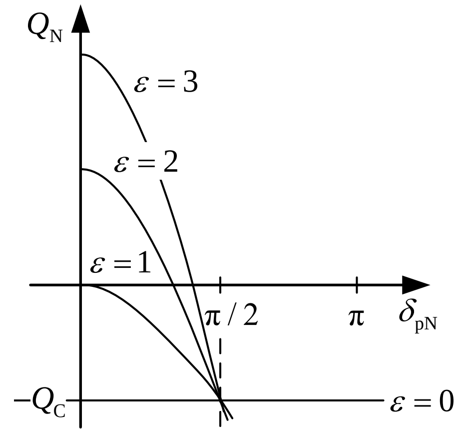
4) The internal grid voltage $\underline{U}_\mathrm{N}$ is a stiff voltage which describes the power exchange within the network and to which the resulting values of the rotor displacement angle and the field ratio are referenced. For $X_\mathrm{V} = 0$, the equations for these variables result in the equations in Section 5.1.

5) In order to avoid negative values for the active and reactive powers occurring during normal operating conditions of synchronous generators when using the PSC, the active and reactive powers consumed at the internal bus of the grid are considered.

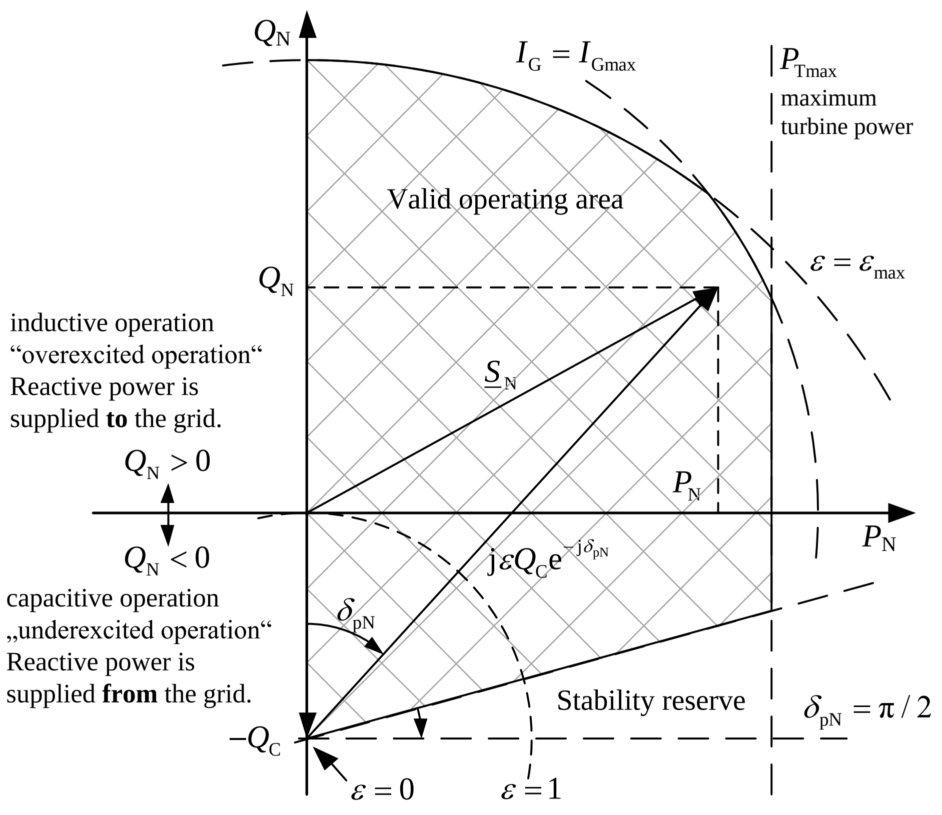
5.4.1 Power to Load Angle Characteristics

Active power to load angle characteristic	Reactive power to load angle characteristic
$P_{\mathrm{N}} = P_{\mathrm{kipp}} \sin \delta_{\mathrm{pN}}$	$Q_{\mathrm{N}} = Q_{\mathrm{C}} \left(\varepsilon \cos \delta_{\mathrm{pN}} - 1 \right)$





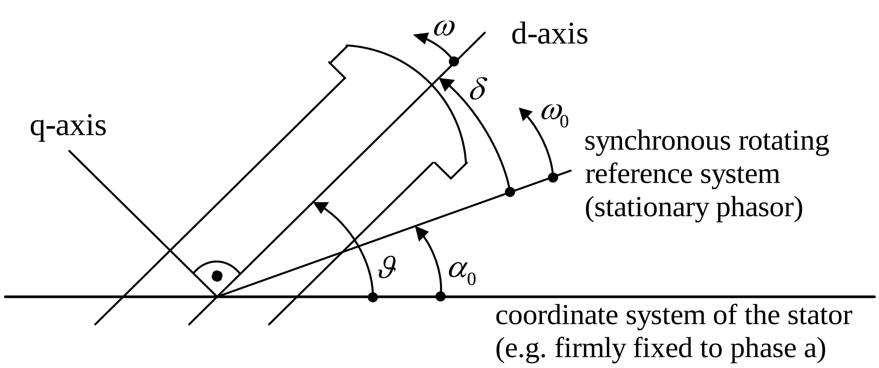
5.4.2 Power Diagram



In practice, the operating ranges inductive and capacitive operation are also referred to as "overexcited operation" and "underexcited operation", although, depending on the current delivered to the grid, capacitive operation with $Q_{\mathrm{N}} < 0$ is also possible in over-excited operation of the synchronous machine with $I_{\mathrm{f}} > I_{\mathrm{f0}}$.

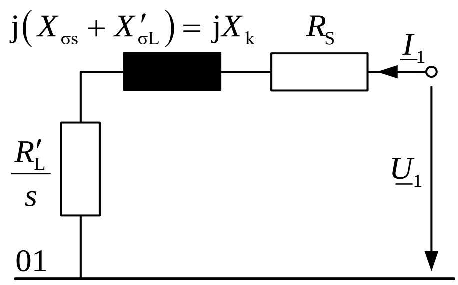
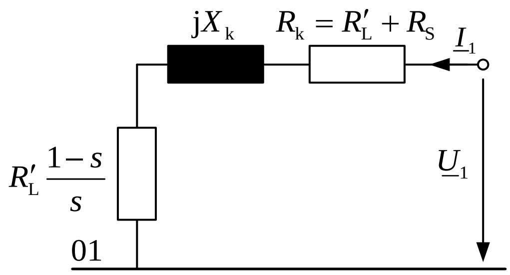
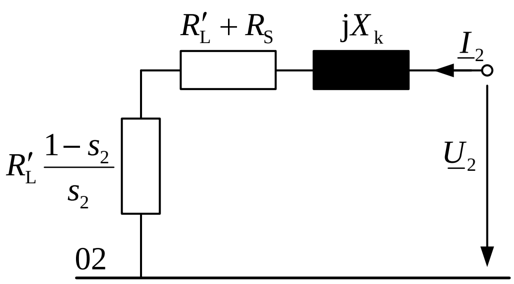
5.5 Principle of Angular Momentum and Equations of Motion

Synchronous speed	$n_0 = \frac{\Omega_0}{2\pi} = \frac{\omega_0}{2\pi p} = \frac{f}{p}$
Number of pole pairs	p
Mechanical and electrical angular velocity	$\Omega = 2\pi n$ and $\omega = \Omega p = 2\pi np$
Principle of angular momentum	$J\dot{\Omega} = M_\mathrm{T} - M_\delta - M_\mathrm{D}$
Moment of inertia	$J = J_\mathrm{rotor} + J_\mathrm{turbine(s)}$
Relation between power, torque and speed	$P = M \cdot \Omega = M \cdot 2\pi n \overset{\Delta\Omega \ll \Omega_0}{\approx} M \cdot \Omega_0$
Equation of motion (for $\Delta\omega \ll \omega_0$)	$\dot{\omega} = \ddot{\delta} = k_\mathrm{M}(P_\mathrm{T} - P_\mathrm{N} - P_\mathrm{D})$ $= k_\mathrm{M}(P_\mathrm{T} - P_\mathrm{N}) - d_\mathrm{M}\Delta\omega$ and $\dot{\delta} = \Delta\omega = \omega - \omega_0 = \dot{\vartheta} - \omega_0$
Damping power	$P_\mathrm{D} = D \cdot \Delta\dot{\delta} = D \cdot \Delta\omega = D(\omega - \omega_0)$
Damping constant	D
Machine constants	$k_\mathrm{M} = \frac{\omega_0}{T_\mathrm{M} S_\mathrm{rG}}$ and $d_\mathrm{M} = D \cdot k_\mathrm{M}$
Electromechanical time constant	$T_\mathrm{M} = \frac{J\Omega_0^2}{S_\mathrm{rG}}$
Relation between angular coordinates	$\vartheta = \omega_0 t + \delta + \alpha_0$



6 Induction Machines with Squirrel-Cage Rotor

6.1 Simplified Equivalent Circuits for Steady-State Operating Conditions

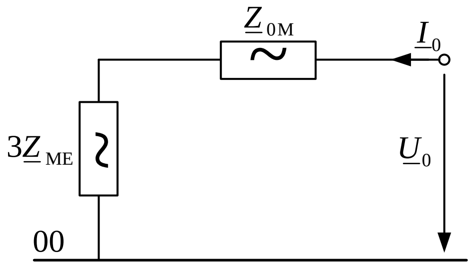
Positive-sequence equivalent circuit (assumption $X_h \to \infty$)	
	
Components of equivalent circuits (rotor quantities converted to stator side)	R_S Stator resistance $X_{\sigma S}$ Stator leakage reactance R'_L Rotor resistance $X'_{\sigma L}$ Rotor leakage reactance
Splitting R'_L / s into slip-dependent and slip-independent parts results in the equivalent circuit on the right side. The stator resistance R_S is negligible for motors in the medium- and high-power range at nominal frequency.	
Slip and synchronous rotational speed	$s = \frac{n_0 - n}{n_0}$ with $n_0 = \frac{f}{p} = \frac{\omega_0}{2\pi p}$
Positive-sequence impedance: $\underline{Z}_1 = \underline{Z}_{1M} = R_S + \frac{R'_L}{s} + \mathrm{j}(X_{\sigma S} + X'_{\sigma L}) \approx \frac{R'_L}{s} + \mathrm{j}X_k$	
Rated apparent power	$S_{rM} = \frac{P_{r\,mech}}{\cos(\varphi_{rM})\,\eta_{rM}}$ (motor operation)
Short-circuit impedance with starting current I_{an} measured when starting with U_{rM} and $s = 1$	$Z_{1M} = \frac{1}{I_{an}/I_{rM}} \frac{U_{rM}^2}{S_{rM}} = \frac{1}{I_{an}/I_{rM}} Z_B$ $= \sqrt{(R_S + R'_L)^2 + X_k^2} \approx \sqrt{R'^2_L + X_k^2}$
Losses at rated current and $s = 1$	$P_{Vkr} = 3(R_S + R'_L) I_{rM}^2 \approx 3 R'_L I_{rM}^2 = R'_L \frac{S_{rM}}{Z_B}$
Negative-sequence equivalent circuit (assumption $X_h \to \infty$)	
Negative-sequence impedance $\underline{Z}_2 = \underline{Z}_{2M} = \frac{R'_L}{2-s} + R_S + \mathrm{j}X_k$ with $s_2 = 2 - s = \frac{-n_0 - n}{-n_0} = \frac{2n_0 - (n_0 - n)}{n_0}$	

Zero-sequence equivalent circuit (assumption $X_\mathrm{h} \to \infty$)

Zero-sequence impedance

$$\underline{Z}_0 = \underline{Z}_{0\mathrm{M}} + 3\underline{Z}_\mathrm{ME} = R'_\mathrm{L} + R_\mathrm{S} + \mathrm{j}X_{0\mathrm{k}} + 3R_\mathrm{ME} + \mathrm{j}3X_\mathrm{ME}$$

with $X_{0\mathrm{k}} < X_\mathrm{k}$



Typically, the neutral points of induction machines are not grounded: $Z_\mathrm{ME} \to \infty$

6.2 Simplified Equivalent Circuits for Transient Operating Conditions

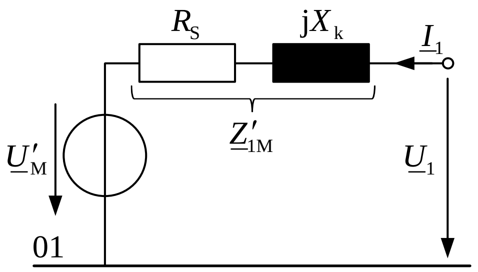
Positive-sequence equivalent circuit (assumption $X_\mathrm{h} \to \infty$)

Transient motor impedance:

$$\underline{Z}'_{1\mathrm{M}} = R_\mathrm{S} + \mathrm{j}X_\mathrm{k} \approx \mathrm{j}X_\mathrm{k}$$

Transient voltage:

$$\underline{U}'_\mathrm{M} = \underline{U}_1(0^-) - \left(R_\mathrm{S} + \mathrm{j}X_\mathrm{k}\right) \underline{I}_1(0^-) = \underline{U}_1(0^-) - \underline{Z}'_{1\mathrm{M}} \, \underline{I}_1(0^-)$$



The transient voltage $\underline{U}'_\mathrm{M}$ is determined using the values of the terminal voltage and current immediately before ($t = 0^-$) the fault (e.g. short-circuit).

6.3 Equation of Motion

Equation of motion (Angular momentum theorem)	$J_\mathrm{M}\dot{\Omega} = M_\mathrm{m} - M_\mathrm{w}$
Resistive torque	$M_\mathrm{w} = M_{\mathrm{w}0} + M_{\mathrm{w}1}\dfrac{\Omega}{\Omega_0} + M_{\mathrm{w}2}\dfrac{\Omega^2}{\Omega_0^2}$
Rotational speed and mechanical angular velocity (f_L = rotor frequency)	$n = \dfrac{f_0 - f_\mathrm{L}}{p} = \dfrac{\Omega}{2\pi} = \dfrac{\Omega_0(1-s)}{2\pi} = \dfrac{\omega_0}{2\pi p}(1-s)$
Torque (Kloss's equation)	$M = M_\mathrm{kipp}\dfrac{2s\, s_\mathrm{kipp}}{s^2 + s_\mathrm{kipp}^2}$
Breakdown torque and slip	$M_\mathrm{kipp} = 3\dfrac{U_1^2}{2\pi n_0}\dfrac{1}{2X_\mathrm{k}}$ and $s_\mathrm{kipp} = \dfrac{R'_\mathrm{L}}{X_\mathrm{k}}$

7 Transformers

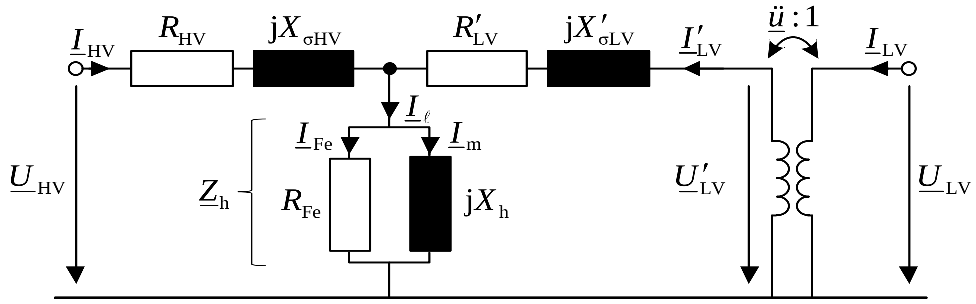
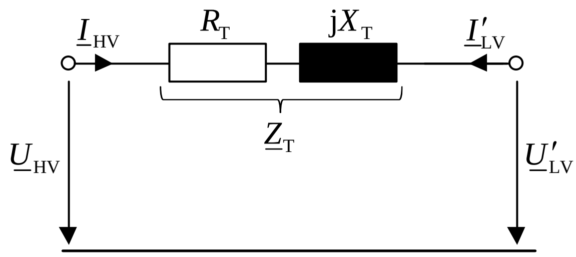
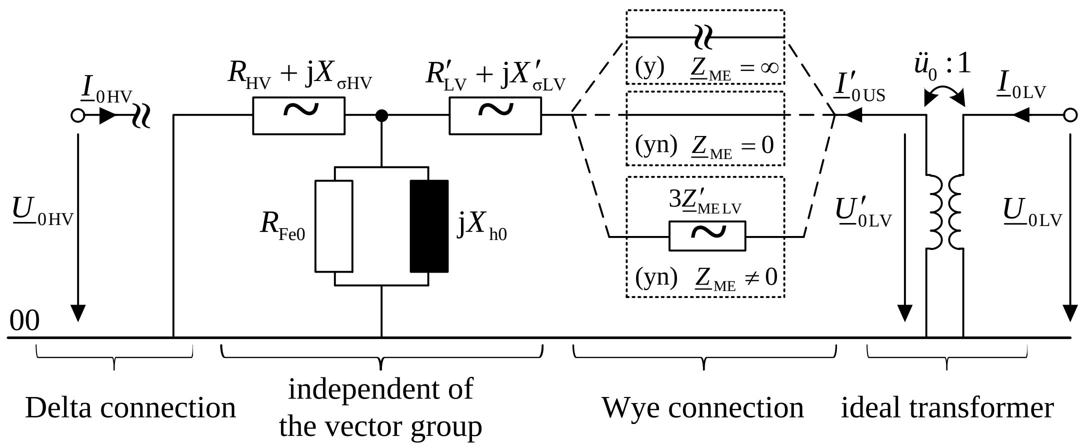
7.1 Vector Group Symbols

Winding connection	Symbol	HV code letter	MV/LV code letter
Wye connection	⅄	Y	y
Delta connection	Δ	D	d
Zigzag connection		Z	z
Grounding available		YN, ZN	yn, zn
Autotransformer		Ya / Yauto	
Vector group symbols of two-winding transformers			
vector group = {HV letter} {LV letter} {vector group code number k}			
Vector group symbols of three-winding transformers			
vector group = {HV letter} {MV letter} {vector group code number $k_{\text{HV-MV}}$} {LV letter} {vector group code number $k_{\text{HV-LV}}$}			
The vector group code number k indicates the multiple of 30° by which the phasors of the phase voltages of the MV and LV winding lag behind those of the HV winding in symmetrical steady-state operation.			

7.2 Conversions of Variables Between Transformer Voltage Levels

Transformation ratios (HV, MV and LV)		
Positive-sequence system	$\underline{\ddot{u}}_1 = \ddot{u}\, e^{jk\frac{\pi}{6}} = \frac{U_{\text{rTHV}}}{U_{\text{rTLV}}}\, e^{jk\frac{\pi}{6}} = \frac{w_{\text{HV}}}{w_{\text{LV}}}\, \underline{m}_{\text{SG}}$	
Negative-sequence system	$\underline{\ddot{u}}_2 = \underline{\ddot{u}}_1^*$	
Zero-sequence system	$\underline{\ddot{u}}_0 = \ddot{u}_0 = \lvert\underline{\ddot{u}}_1\rvert = \ddot{u}$	
Conversion of pos.-seq. variables	LV to HV	HV to LV
Voltage	$\underline{U}'_{1\text{LV}} = \underline{\ddot{u}}_1\, \underline{U}_{1\text{LV}}$	$\underline{U}'_{1\text{HV}} = \frac{1}{\underline{\ddot{u}}_1}\, \underline{U}_{1\text{HV}}$
Current	$\underline{I}'_{1\text{LV}} = \frac{1}{\underline{\ddot{u}}_1^*}\, \underline{I}_{1\text{LV}}$	$\underline{I}'_{1\text{HV}} = \underline{\ddot{u}}_1^*\, \underline{I}_{1\text{HV}}$
Impedance	$\underline{Z}'_{1\text{LV}} = \ddot{u}_1^2\, \underline{Z}_{1\text{LV}}$	$\underline{Z}'_{1\text{HV}} = \frac{1}{\ddot{u}_1^2}\, \underline{Z}_{1\text{HV}}$
Corresponding conversions of negative- and zero-sequence variables are carried out analogously using the respective transformation ratios.		

7.3 Two-Winding Transformer

<table>
<tr><th colspan="2">Positive- ($\underline{\ddot{u}} = \underline{\ddot{u}}_1$) and negative-sequence ($\underline{\ddot{u}} = \underline{\ddot{u}}_2$) equivalent circuits</th></tr>
<tr><td colspan="2">Complete equivalent circuit [6] (with ideal transformer [7]) related to the HV level):

</td></tr>
<tr><td>Magnetizing impedance</td><td>$\underline{Z}_h = R_{Fe} \parallel jX_h$</td></tr>
<tr><td colspan="2">Simplified equivalent circuit (without ideal transformer – related to the HV level):
(assuming $\underline{Z}_h \rightarrow \infty$)

</td></tr>
<tr><td>Short-circuit impedance</td><td>$\underline{Z}_T = R_T + jX_T = R_{HV} + R'_{LV} + j(X_{\sigma HV} + X'_{\sigma LV})$</td></tr>
<tr><th colspan="2">Zero-sequence equivalent circuit</th></tr>
<tr><td>The zero-sequence equivalent circuit depends on</td><td>1) vector group
2) neutral point grounding
3) type of transformer core construction</td></tr>
<tr><td colspan="2">T–-equivalent circuit with ideal transformer – related to the HV level – Dy(n)5:

</td></tr>
</table>

[6] Since the elements of the positive- and negative-sequence equivalent circuits are equal, no corresponding index was used. However, the different transformation ratios have to be considered (see Section 7.2).

[7] See Section 2.7.3.

7.4 Calculation of the Transformer Equivalent Circuit Elements

7.4.1 Calculation of the Series Impedance Using the Short-Circuit Test

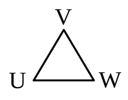
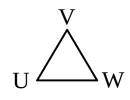
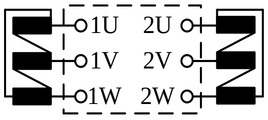
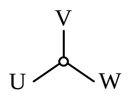
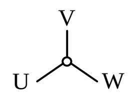
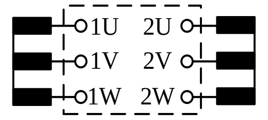
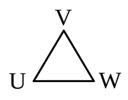
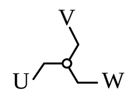
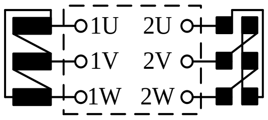
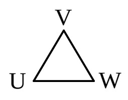
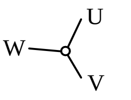
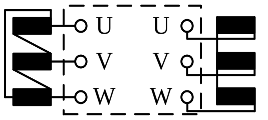
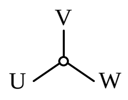
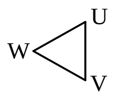
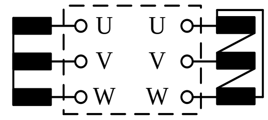
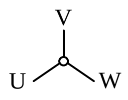
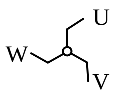
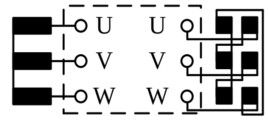
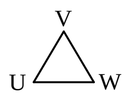
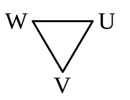
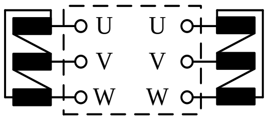
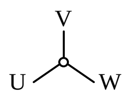
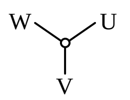
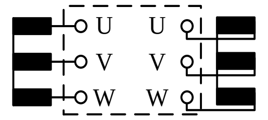
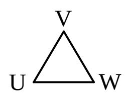
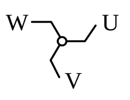
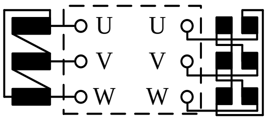
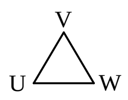
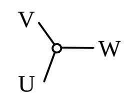
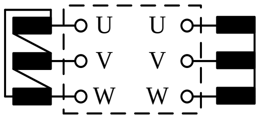
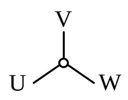
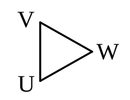
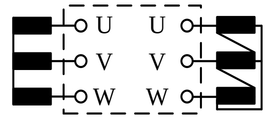
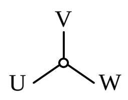
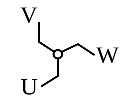
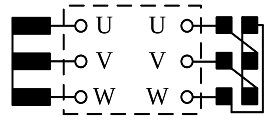
The magnetizing impedance can be neglected while calculating the series impedance.	
Reference impedance	$Z_B = \frac{U_{rT}^2}{S_{rT}}$ $U_{rT} = U_{rTHV}$ for the HV level; $U_{rT} = U_{rTLV}$ for the LV level
Short-circuit impedance	$Z_T = u_k \cdot U_{rT}^2 / S_{rT} = u_k Z_B$
Resistance of the short-circuit impedance	$R_T = r_T Z_B = r_T \frac{U_{rT}^2}{S_{rT}} = \frac{P_{Vkr}}{3 I_{rT}^2} = \frac{P_{Vkr}}{S_{rT}} Z_B = u_R Z_B$
Reactance of the short-circuit impedance	$X_T = \sqrt{Z_T^2 - R_T^2} = \sqrt{u_k^2 - u_R^2}\, Z_B = u_X Z_B$
Generally, R_T and X_T are equally divided between the HV and LV winding impedances[8] of the positive-, negative- and zero-sequence equivalent circuits, see section 7.3.	

7.4.2 Calculation of the Magnetizing Impedance by an Open-Circuit Test

The series impedance can be neglected for the calculation of the magnetizing impedance.	
Open-circuit current	$I_\ell \approx \frac{U_{rT}}{\sqrt{3} Z_h} \quad \rightarrow i_\ell = \frac{1}{Z_h} \frac{U_{rT}^2}{S_{rT}} = \frac{1}{Z_h} Z_B$
Magnetizing current	$I_m \approx \frac{U_{rT}}{\sqrt{3} X_h} \quad \rightarrow i_m = \frac{1}{X_h} \frac{U_{rT}^2}{S_{rT}} = \frac{1}{X_h} Z_B$
Open-circuit impedance	$Z_h = \frac{1}{i_\ell} \frac{U_{rT}^2}{S_{rT}} = \frac{1}{i_\ell} Z_B$
Iron loss resistance	$R_{Fe} = \frac{U_{rT}^2}{P_{V\ell r}} = \frac{1}{P_{V\ell r} / S_{rT}} Z_B$
Magnetizing reactance	$X_h = \frac{Z_h R_{Fe}}{\sqrt{R_{Fe}^2 - Z_h^2}} = \frac{1}{i_m} \frac{U_{rT}^2}{S_{rT}} = \frac{1}{i_m} Z_B$
Zero-sequence magnetizing reactance	$X_{h0} = k_0 X_{h1}$ (k_0 dependent on core construction)
The elements of the zero-sequence equivalent circuit can either be determined by an additional open-circuit test by feeding into a zero-sequence system or in a simplified manner by using a factor k_0 to estimate the elements.	

8) The impedance is specified as impedance per phase for the three-phase transformers. Exemplarily, the impedances of delta-connected windings are converted to an equivalent representation of wye-connected windings.

7.5 Typical Vector Groups (DIN VDE 0532)

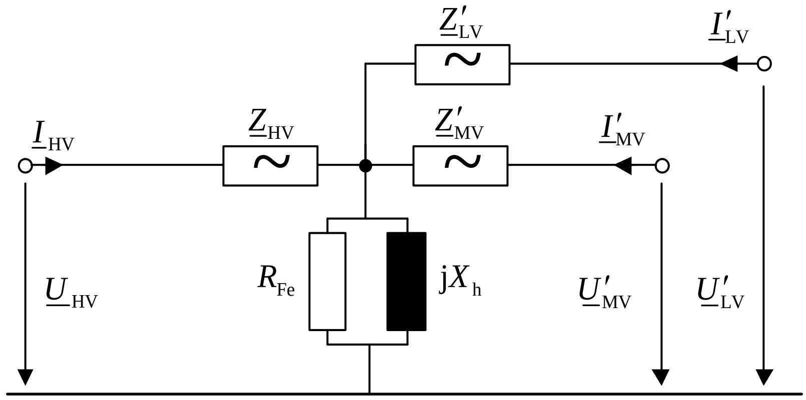
Code number	Vector group	Phasor diagram HV	LV	Circuit diagram HV LV
0	Dd0			
	Yy0			
	Dz0			
5	**Dy5**			
	Yd5			
	Yz5			
6	Dd6			
	Yy6			
	Dz6			
11	Dy11			
	Yd11			
	Yz11			

Yy0 Indicates preferred vector groups

7.6 Three-Winding Transformer

Positive- and negative-sequence equivalent circuits

Simplified equivalent circuit (without an ideal transformer, related to the HV level):



The calculation of the equivalent circuit parameters of a three-winding transformer is subject to the same calculation specifications as for the two-winding transformer. A minimum of three short-circuit tests is required to calculate the three series impedances. The magnetizing impedance (X_h and R_{Fe}) is calculated according to Section 7.4.2 using the data of the open-circuit test.

Maximum power transport between windings	$S_{rTHVMV} = \min\left(S_{rTHV}, S_{rTMV}\right)$
	$S_{rTHVLV} = \min\left(S_{rTHV}, S_{rTLV}\right)$
	$S_{rTMVLV} = \min\left(S_{rTMV}, S_{rTLV}\right)$
Short-circuit impedances related to the HV level	$Z_{HVMV} = u_{kHVMV} \frac{U_{rTHV}^2}{S_{rTHVMV}}$
	$Z_{HVLV} = u_{kHVLV} \frac{U_{rTHV}^2}{S_{rTHVLV}}$
	$Z'_{MVLV} = u_{kMVLV} \frac{U_{rTHV}^2}{S_{rTMVLV}}$
Winding impedances [9)]	$\begin{bmatrix} \underline{Z}_{HV} \\ \underline{Z}'_{MV} \\ \underline{Z}'_{LV} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \underline{Z}_{HVMV} \\ \underline{Z}_{HVLV} \\ \underline{Z}'_{MVLV} \end{bmatrix}$

9) The values of the winding impedances can partly become negative. This is due to the structure of the equivalent circuit used.

8 Transmission Lines

The equations for the line constants and the positive-, negative- and zero-sequence equivalent circuits are identical with regard to the structure. By means of the respective primary line parameters per unit length for the positive-, negative- and zero-sequence system, the line constants are calculated and the elements of the equivalent circuits are parameterized.

8.1 Surge Impedance and Propagation Constant

Transmission lines (affected by losses)	
Characteristic impedance (surge impedance)	$\underline{Z}_\mathrm{w} = \sqrt{\dfrac{R' + \mathrm{j}\omega L'}{G' + \mathrm{j}\omega C'}} = \dfrac{1}{\underline{Y}_\mathrm{w}}$
Propagation constant with attenuation constant α and phase(change) constant β	$\underline{\gamma} = \sqrt{(R' + \mathrm{j}\omega L')(G' + \mathrm{j}\omega C')} = \alpha + \mathrm{j}\beta$
R', L', G' and C' are the primary line parameters per unit length in Ω / km, H / km, S / km and F / km	
Low-loss transmission lines ($R' \ll \omega L'$ and $G' \ll \omega C'$)	
Characteristic impedance[10] and characteristic admittance	$\underline{Z}_\mathrm{w} = \dfrac{1}{\underline{Y}_\mathrm{w}} \approx \sqrt{\dfrac{L'}{C'}}\left(1 - \mathrm{j}\dfrac{1}{2}\dfrac{R'}{\omega L'}\right) \approx \sqrt{\dfrac{L'}{C'}}$
Propagation constant with attenuation constant α and phase(change) constant β	$\underline{\gamma} = \dfrac{1}{2}\left(\dfrac{R'}{L'} + \dfrac{G'}{C'}\right)\sqrt{L'C'} + \mathrm{j}\omega\sqrt{L'C'}$

8.2 Solution of the Line Equations in the Frequency Domain

Voltage and current at the location x along the line (node A: x = 0, node B: x = l)
depending on the terminal values at node A: $$\begin{bmatrix} \underline{U}(x) \\ \underline{I}(x) \end{bmatrix} = \begin{bmatrix} \cosh(\underline{\gamma}\, x) & -\underline{Z}_\mathrm{w} \sinh(\underline{\gamma}\, x) \\ \underline{Y}_\mathrm{w} \sinh(\underline{\gamma}\, x) & -\cosh(\underline{\gamma}\, x) \end{bmatrix} \begin{bmatrix} \underline{U}_\mathrm{A} \\ \underline{I}_\mathrm{A} \end{bmatrix}$$
depending on the terminal values at node B: $$\begin{bmatrix} \underline{U}(x) \\ \underline{I}(x) \end{bmatrix} = \begin{bmatrix} \cosh(\underline{\gamma}(l-x)) & -\underline{Z}_\mathrm{w} \sinh(\underline{\gamma}(l-x)) \\ \underline{Y}_\mathrm{w} \sinh(\underline{\gamma}(l-x)) & -\cosh(\underline{\gamma}(l-x)) \end{bmatrix} \begin{bmatrix} \underline{U}_\mathrm{B} \\ \underline{I}_\mathrm{B} \end{bmatrix}$$

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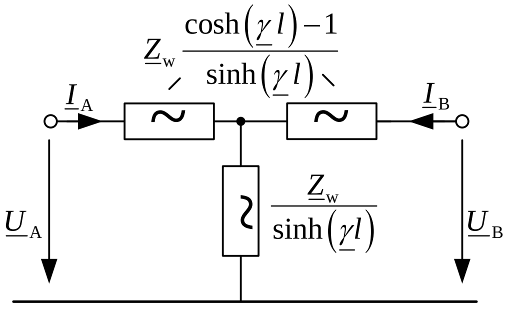
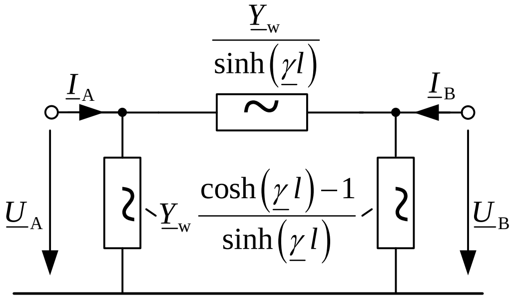
[10] The capacitive component of the characteristic impedance (imaginary part) can usually be neglected for low-loss lines due to its small size.

8.3 Distributed Line Model and Two-Port Equations

Equivalent circuit with distributed parameters



T–equivalent circuit



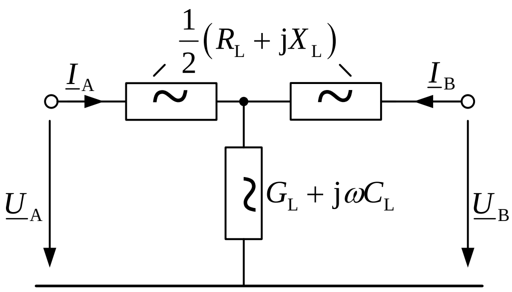
Π–equivalent circuit

Two-port equations

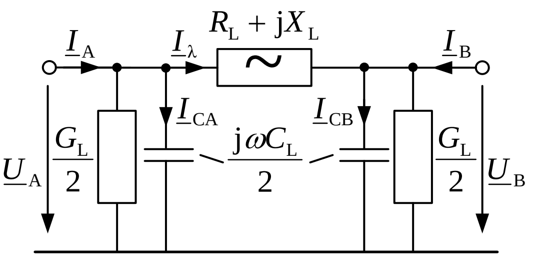
Impedance representation	$\begin{bmatrix}\underline{U}_A\\ \underline{U}_B\end{bmatrix}=\begin{bmatrix}\underline{Z}_w\dfrac{\cosh(\underline{\gamma} l)}{\sinh(\underline{\gamma} l)} & \underline{Z}_w\dfrac{1}{\sinh(\underline{\gamma} l)}\\ \underline{Z}_w\dfrac{1}{\sinh(\underline{\gamma} l)} & \underline{Z}_w\dfrac{\cosh(\underline{\gamma} l)}{\sinh(\underline{\gamma} l)}\end{bmatrix}\begin{bmatrix}\underline{I}_A\\ \underline{I}_B\end{bmatrix}$
Admittance representation	$\begin{bmatrix}\underline{I}_A\\ \underline{I}_B\end{bmatrix}=\begin{bmatrix}\underline{Y}_w\dfrac{\cosh(\underline{\gamma} l)}{\sinh(\underline{\gamma} l)} & -\underline{Y}_w\dfrac{1}{\sinh(\underline{\gamma} l)}\\ -\underline{Y}_w\dfrac{1}{\sinh(\underline{\gamma} l)} & \underline{Y}_w\dfrac{\cosh(\underline{\gamma} l)}{\sinh(\underline{\gamma} l)}\end{bmatrix}\begin{bmatrix}\underline{U}_A\\ \underline{U}_B\end{bmatrix}$
Iterative representation (cascade or transmission representation)	$\begin{bmatrix}\underline{U}_A\\ \underline{I}_A\end{bmatrix}=\begin{bmatrix}\cosh(\underline{\gamma} l) & -\underline{Z}_w\sinh(\underline{\gamma} l)\\ \underline{Y}_w\sinh(\underline{\gamma} l) & -\cosh(\underline{\gamma} l)\end{bmatrix}\begin{bmatrix}\underline{U}_B\\ \underline{I}_B\end{bmatrix}$

8.4 Approximations for Electrically Short Transmission Lines ($|\underline{\gamma} l| << 1$)

Equivalent circuit with lumped parameters



T– equivalent circuit



Π – equivalent circuit

Equivalent circuit elements (Line length l)	$R_L = R'l$, $L_L = L'l$, $C_L = C'l$, $G_L = G'l$

See Section 2.7.2 for two-port equations of T– and Π–equivalent circuit

8.5 Operational Performance

Voltage drop	$\lvert\Delta \underline{U}_{\mathrm{Z}}\rvert = \lvert\underline{U}_{\mathrm{A}} - \underline{U}_{\mathrm{B}}\rvert$
Voltage difference	$\Delta U = \lvert\underline{U}_{\mathrm{A}}\rvert - \lvert\underline{U}_{\mathrm{B}}\rvert$
Transmission angle	$\delta_{\mathrm{AB}} = \delta_{\mathrm{A}} - \delta_{\mathrm{B}}$
Capacitive charging current[11]	$\underline{I}_{\mathrm{C}} = \underline{I}_{\mathrm{A}}\big\rvert_{\underline{I}_{\mathrm{B}}=0} \approx \underline{I}_{\mathrm{CA}} + \underline{I}_{\mathrm{CB}} = \mathrm{j}\omega \frac{C_{\mathrm{L}}}{2}\left(\underline{U}_{\mathrm{A}} + \underline{U}_{\mathrm{B}}\right)$
Capacitive charging power[12]	$Q_{\mathrm{C}} = \mathrm{Im}\left\{3\underline{U}_{\mathrm{A}}\,\underline{I}_{\mathrm{C}}^{*}\right\} \approx \omega C_{\mathrm{L}}\,U_{\mathrm{nN}}^{2}$

8.6 Terminal Power, Losses and Reactive Power Demand

Power at the line terminals (see two-port network in 2.7.2)	$\begin{bmatrix}\underline{S}_{\mathrm{A}} \\ \underline{S}_{\mathrm{B}}\end{bmatrix} = \begin{bmatrix}P_{\mathrm{A}} + \mathrm{j}Q_{\mathrm{A}} \\ P_{\mathrm{B}} + \mathrm{j}Q_{\mathrm{B}}\end{bmatrix} = 3\begin{bmatrix}\underline{Y}_{\mathrm{AA}}^{*}U_{\mathrm{A}}^{2} + \underline{Y}_{\mathrm{AB}}^{*}\underline{U}_{\mathrm{A}}\underline{U}_{\mathrm{B}}^{*} \\ \underline{Y}_{\mathrm{BB}}^{*}U_{\mathrm{B}}^{2} + \underline{Y}_{\mathrm{BA}}^{*}\underline{U}_{\mathrm{B}}\underline{U}_{\mathrm{A}}^{*}\end{bmatrix}$
Transmissible active power (for electrically short lines with neglect of R', G' and C')	$P_{\mathrm{A}} = -P_{\mathrm{B}} = P_{\mathrm{AB}} \approx 3\frac{U_{\mathrm{A}}U_{\mathrm{B}}}{X_{\mathrm{L}}}\sin\delta_{\mathrm{AB}}$
Line losses P_{V} and reactive power demand Q_{V}	$\underline{S}_{\mathrm{V}} = P_{\mathrm{V}} + \mathrm{j}Q_{\mathrm{V}} = \underline{S}_{\mathrm{A}} + \underline{S}_{\mathrm{B}}$
Line losses (example of Π-equivalent circuit with lumped parameters)	$P_{\mathrm{V}} = 3R_{\mathrm{L}}\,I_{\lambda}^{2} + 3\frac{1}{2}G_{\mathrm{L}}\left(U_{\mathrm{A}}^{2} + U_{\mathrm{B}}^{2}\right) = P_{\mathrm{A}} + P_{\mathrm{B}}$
Reactive power demand (example of Π-equivalent circuit with lumped parameters)	$Q_{\mathrm{V}} = 3\omega L_{\mathrm{L}}\,I_{\lambda}^{2} - 3\frac{1}{2}\omega C_{\mathrm{L}}\left(U_{\mathrm{A}}^{2} + U_{\mathrm{B}}^{2}\right) = Q_{\mathrm{A}} + Q_{\mathrm{B}}$

8.7 Surge Impedance Loading (Natural Load)

Surge impedance loading (SIL)	
Condition for surge impedance loading	$\underline{Z}_{\mathrm{B}} = \underline{Z}_{\mathrm{w}} \rightarrow \underline{U}_{\mathrm{B}} = -\underline{Z}_{\mathrm{w}} \cdot \underline{I}_{\mathrm{B}}$ (Termination at node B with characteristic impedance)
Natural load[12]	$\underline{S}_{\mathrm{Nat}} = 3\frac{U_{\mathrm{B}}^{2}}{\underline{Z}_{\mathrm{w}}^{*}} \approx P_{\mathrm{Nat}} = 3\frac{U_{\mathrm{B}}^{2}}{Z_{\mathrm{w}}}$ (apparent power output at terminal B)
Above natural load[11]	$Q_{\mathrm{V}} > 0$ for $P_{\mathrm{B}} > P_{\mathrm{Nat}}$ resp. $Z_{\mathrm{B}} < Z_{\mathrm{w}}$
Below natural load[11]	$Q_{\mathrm{V}} < 0$ for $P_{\mathrm{B}} < P_{\mathrm{Nat}}$ resp. $Z_{\mathrm{B}} > Z_{\mathrm{w}}$

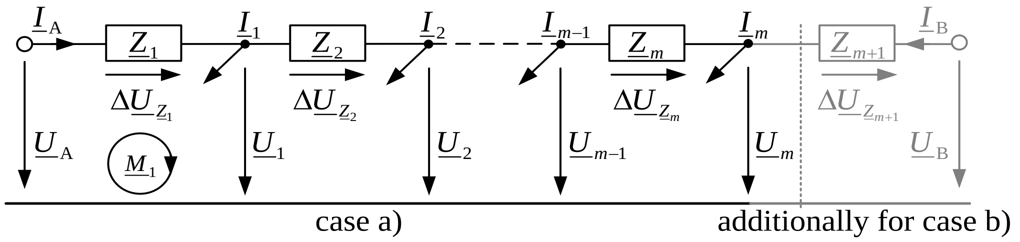
---

[11] Approximation applies to Π-equivalent circuit with lumped parameters.

[12] Approximation applies to low-loss lines (see Section 8.1)

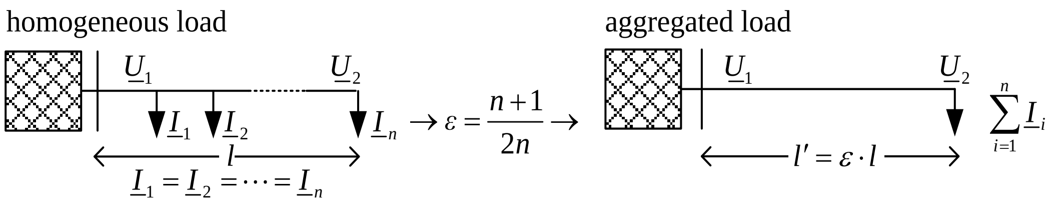
9 Medium- and Low-Voltage Grids

9.1 Current Distribution

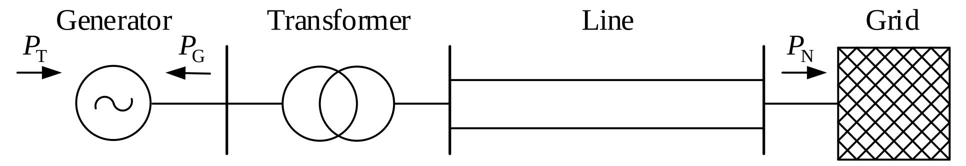
General equivalent circuit: Applies only to voltage-independent load currents	
	
Case a) Single-fed line: $\underline{U}_A$ given, without $\underline{Z}_{m+1}$	
Kirchhoff's voltage law provides *m* independent equations	$\underline{U}_1 = \underline{U}_A - \underline{Z}_1 \underbrace{(\underline{I}_1 + \underline{I}_2 + \cdots + \underline{I}_m)}_{\underline{I}_A}$ (1) $\underline{U}_2 = \underline{U}_1 - \underline{Z}_2 (\quad \underline{I}_2 + \cdots + \underline{I}_m)$ (2) $\vdots$ $\vdots$ $\underline{U}_B = \underline{U}_i - \underline{Z}_m \underline{I}_m$ (m)
Case b1) Double-fed line: $\underline{U}_A = \underline{U}_B$	
General mesh A – B	$\underline{U}_A - \underline{U}_B = \sum_{i=1}^{m+1} \Delta\underline{U}_{Z_i} = \underline{Z}_1 (\underline{I}_1 + \underline{I}_2 + \cdots - \underline{I}_B)$ $+ \underline{Z}_2 (\quad \underline{I}_2 + \cdots - \underline{I}_B) + \ldots + \underline{Z}_{m+1}(-\underline{I}_B)$
Calculation of one terminal current using "torque approach" (for node B or A)	$\underline{I}_B = \frac{\underline{Z}_1 \underline{I}_1 + (\underline{Z}_1 + \underline{Z}_2)\underline{I}_2 + \cdots + (\underline{Z}_1 + \underline{Z}_2 + \cdots + \underline{Z}_m)\underline{I}_m}{\underline{Z}_1 + \underline{Z}_2 + \cdots + \underline{Z}_{m+1}}$ Stepwise determination of the current distribution
Case b2) Double-fed line: $\underline{U}_A \neq \underline{U}_B$	
General approach	1.) Calculation of a preliminary current distribution with $\underline{U}_A = \underline{U}_B$ corresponding to case b1 2.) Superposition with an offset current $\Delta\underline{I}_{AB} = \frac{\underline{U}_A - \underline{U}_B}{\sum_{i=1}^{m+1} \underline{Z}_i}$
The current distribution can be used to determine the voltage drops $\Delta\underline{U}_{Z_i}$.	

9.2 Load Application Factor ε

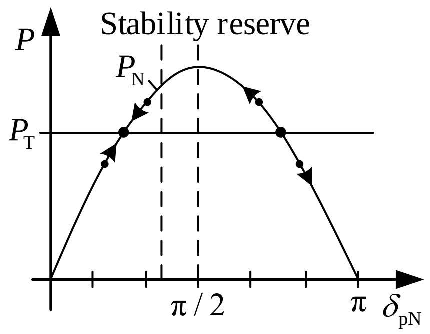
Conversion of a uniformly distributed load into a concentrated load



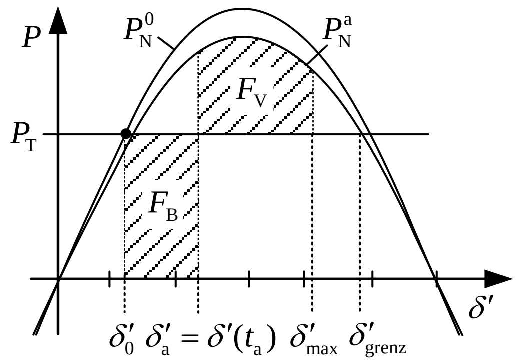
10 Rotor Angle Stability of the Single-Machine Problem



10.1 Small-Signal Stability

Equation of motion (see 5.5)	$\dot{\omega} = k_M (P_T - P_N)$ and $\dot{\delta} = \omega - \omega_0 = \dot{\vartheta} - \omega_0$
Operating point with turbine power P_T (losses are neglected)	
Requirement for small-signal stability	$\frac{dP_N}{d\delta_{pN}} > 0$ resp. $-\frac{\pi}{2} < \delta_{pN} < \frac{\pi}{2}$
Synchronizing power	$P_s = \frac{dP_N}{d\delta_{pN}} = P_{kipp} \cos \delta_{pN}$

10.2 Transient Stability

Operating points with turbine power P_T (losses are neglected) Applies to transient quantities: $\delta' = \varphi_{U'} - \varphi_{UN}$ and (transient voltage $\underline{U}'$ see 5.2) $P_N^a = 3 \frac{U' U_N}{X'_d + X_V} \sin \delta'$	 (index 0: before fault, index a: after fault clearance)
Requirement for transient stability	$F_V = F_B$ (Law of equal areas)
Law of equal areas maximum fault clearance time	$F_{V\max} = F_B = -\int_{\delta'_{a\max}}^{\delta'_{max} = \delta'_{grenz}} (P_T - P_{kipp}^a \sin \delta') \, d\delta'$
Maximum fault clearance time	$t_{a\max} = \sqrt{\frac{2}{k_M P_T} \left(\delta'_{a\max} - \delta'_0 \right)}$

11 Line Frequency Control

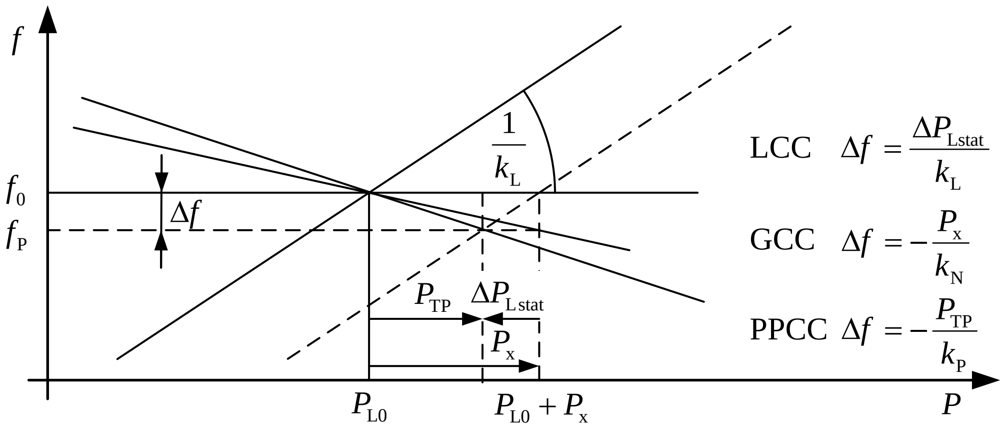
11.1 Balance Model of the Power System

Differential equation for the calculation of the line frequency at $P_{T0} = P_{L0}$	$M_N \frac{\Delta\dot{f}}{f_0} = \Delta P_T - \Delta P_{Lx} = P_{TP} + P_{TS} - \Delta P_{Lstat} - P_x$
Turbine and load power at the operating point	P_{T0} and P_{L0}
Inertia constant of the power system	$M_N = P_G\,T_G + P_M\,T_M$
Total rated active power of synchronous generators in operation	$P_G = \sum_{i=1}^{m_G} P_{rGi}$
Total rated active power of induction motors in operation	$P_M = \sum_{i=1}^{m_M} P_{rMi}$
Equivalent generator time constant	$T_G = \frac{1}{P_G}\sum_{i=1}^{m_G} J_{Gi}\,\Omega_0^2$
Equivalent motor time constant	$T_M = \frac{1}{P_M}\sum_{i=1}^{m_M} J_{Mi}\,\Omega_0^2$

11.2 Proportional Gains and Droops

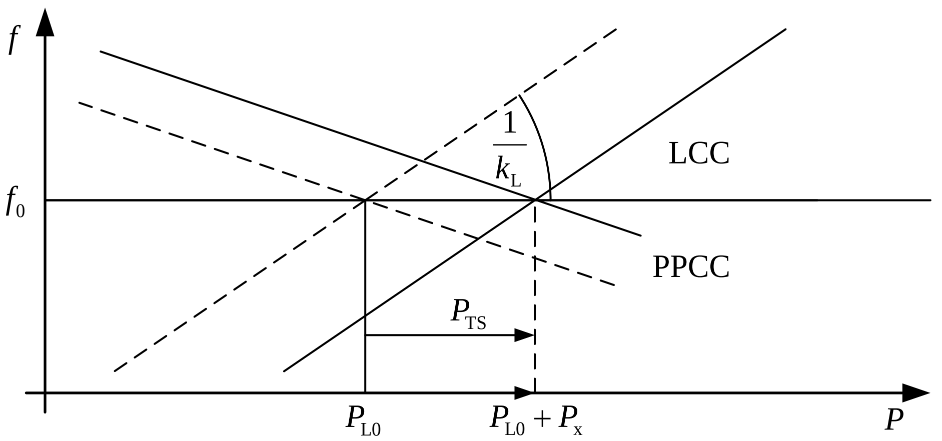
Frequency bias of primary control (absolute and per unit value) and control droop of the generator *i*	$k_{Pi} = -\frac{\Delta P_{Ti}}{\Delta f}$, $k'_{Pi} = k_{Pi}\frac{f_0}{P_{rGi}}$ and $s_{Pi} = \frac{1}{k'_{Pi}} = \frac{1}{k_{Pi}}\frac{P_{rGi}}{f_0}$
Total frequency bias of primary control and total control droop of generators	$k_P = \sum_{i=1}^{m_G} k_{Pi} = \frac{1}{f_0}\sum_{i=1}^{m_G}\frac{P_{rGi}}{s_{Pi}}$ and $s_P = P_G\left(\sum_{i=1}^{m_G}\frac{P_{rGi}}{s_{Pi}}\right)^{-1} = \frac{1}{k_P}\frac{P_G}{f_0}$
Frequency bias (absolute and per unit value) and droop of the load *i*	k_{Li} , $k'_{Li} = k_{Li}\frac{f_0}{P_{L0i}}$ and $s_{Li} = \frac{1}{k'_{Li}} = \frac{1}{k_{Li}}\frac{P_{L0i}}{f_0}$
Total load frequency bias and total load droop	$k_L = \left.\frac{dP_{Lstat}}{df}\right\vert_{AP} = \sum_{i=1}^{m_L} k_{Li}$ and $s_L = P_{L0}\cdot\left(\sum_{i=1}^{m_L}\frac{P_{L0i}}{s_{Li}}\right)^{-1} = \frac{1}{k_L}\frac{P_{L0}}{f_0}$
Frequency bias of the power system	$k_N = k_P + k_L$

11.3 Power System Operating Point after Completion of Primary Control (without Secondary Control)



Unplanned change of load	P_x
Steady-state operation (after completion of primary control)	$\Delta \dot{f} = 0$
Primary control power (Power Plant Characteristic Curve (PPCC))	$\Delta P_T = P_{TP} = -k_P \, \Delta f$
Frequency-dependent change of load (Load Characteristic Curve (LCC))	$\Delta P_{Lstat} = k_L \, \Delta f$
Load change in the perturbed power system	$\Delta P_{Lx} = \Delta P_{Lstat} + P_x$
Frequency change in power system (Grid Characteristic Curve (GCC))	$\Delta f = -\frac{P_x}{k_N}$

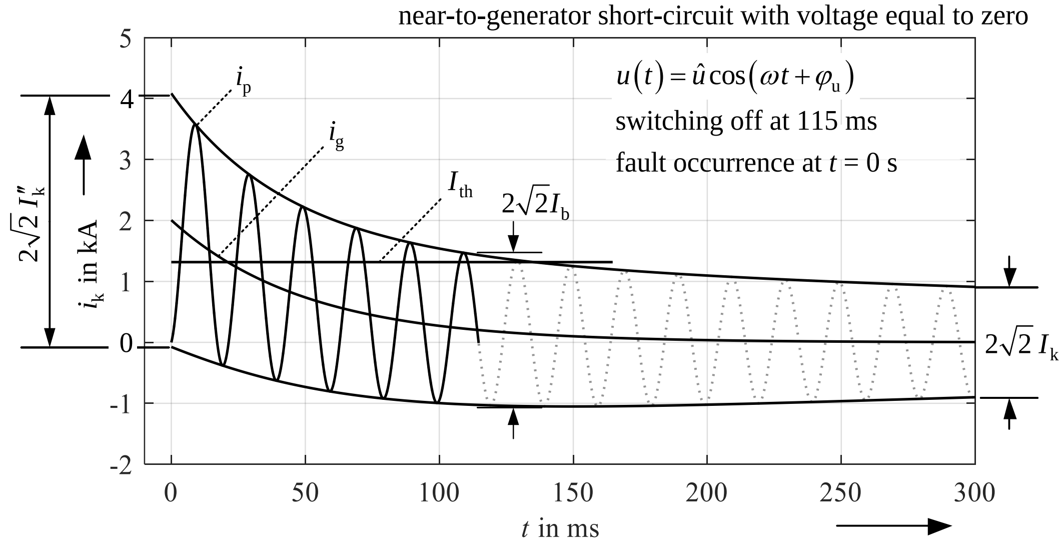
11.4 Power System Operating Point after Completion of Secondary Control



Total secondary control power activation	$\Delta P_T = P_{TS} = P_x$

12 Short-Circuit Current Calculation

12.1 Short-Circuit Current Over Time



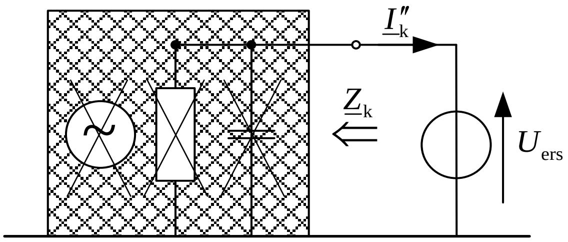
$$i_k(t) \approx \sqrt{2}\left[(I''_k - I'_k)\,e^{-\frac{t}{T''_d}} + (I'_k - I_k)\,e^{-\frac{t}{T'_d}} + I_k\right]\cos(\omega t + \alpha) - \sqrt{2}I''_k \cos\alpha\, e^{-\frac{t}{T_g}}$$

Near-to-generator short-circuit if one synchronous machine contributes to an initial short-circuit current $I''_{kGi} > 2I_{rGi}$	$I''_k > I_b > I_k$
Far-from-generator short-circuit	$I''_k \approx I_b \approx I_k$

12.2 Characteristic Short-Circuit Current Parameters

Initial symmetrical short-circuit current (rms value of the AC symmetrical component applicable at instant of short-circuit)	I''_k
Transient short-circuit current (rms value of the AC symmetrical component in transient time domain)	I'_k
Steady-state short-circuit current (rms value of the short-circuit current after the decay of the transient phenomena)	I_k
Peak short-circuit current (maximal possible instantaneous value of the prospective short-circuit current)	i_p
Symmetrical short-circuit breaking current (rms value of the AC symmetrical component at the instant of contact separation)	I_b
Thermal equivalent short-circuit current (rms value of short-circuit current having the same thermal effect and the same duration T_k as the decaying short-circuit current)	I_{th}
Time constants (sub-transient, transient, direct-current)	T''_d, T'_d, T_g
Angle of short-circuit impedance $\underline{Z}_k$ at short-circuit location	φ_{Zk}
Initial phase angle of the voltage at short-circuit location	φ_u
Initial phase angle of the short-circuit current $i_k(t)$	$\alpha = \varphi_u - \varphi_{Zk}$

12.3 Method of the Equivalent Voltage Source at the Short-Circuit Location (According to IEC 60909 and VDE 0102)

Calculation independent of operating point, estimation of the minimal and maximal absolute value of the initial symmetrical short-circuit current I''_{k} [13]	
	• Reverse inject into the passive network at the short-circuit location • Equivalent voltage source U_{ers} at the short-circuit location • Neglect of shunt admittances and non-rotating loads[14]
Initial symmetrical short-circuit current	$I''_{k} = \frac{U_{ers}}{Z_{k}} = c\frac{U_{n}}{\sqrt{3}Z_{k}}$
Equivalent voltage source at the short-circuit location	$U_{ers} = c\frac{U_{n}}{\sqrt{3}}$
Voltage factor *c* for the calculation of the maximum short-circuit currents[15]	$c_{max} = 1{,}1$
Voltage factor *c* for the calculation of the minimum short-circuit currents[15]	$c_{min} = 1{,}0$
Short-circuit impedance at the short-circuit location[16] (effective impedance at the instant of the short-circuit)	$\underline{Z}_{k} = R_{k} + \mathrm{j}X_{k}$
Peak short-circuit current (κ: see factor in Section 12.5.1)	$i_{p} = \kappa\sqrt{2}\,I''_{k}$
Symmetrical short-circuit breaking current[17] (*μ*: see factor in Section 12.5.2)	$I_{b} = \mu\,I''_{k}$
Thermal equivalent short-circuit current (*m* and *n*: see factors in Section 12.5.3)	$I_{th} = \sqrt{m+n}\,I''_{k}$
Steady state short-circuit current	I_{k}

[13] For the consideration of the short-circuit current contributions of doubly-fed induction generator-based wind turbines and power station units with full-size converter see IEC 60909 and section 12.4.

[14] Deviations are possible for the zero-sequence system (see IEC 60909).

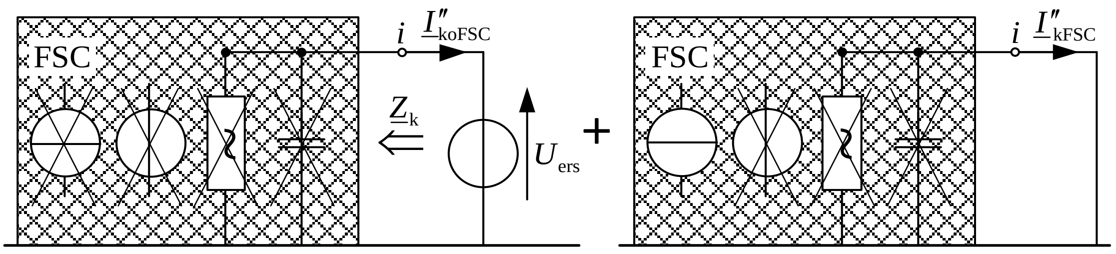
[15] Valid for nominal system voltages > 1 kV

[16] For the calculation of the maximum initial symmetrical short-circuit current according to IEC 60909, impedance correction factors shall be applied acc. section 12.6.

[17] Only valid for near-to-generator single-fed three-phase short-circuits.

12.4 Extending the Method of the Equivalent Voltage Source at the Short-Circuit Location from Section 12.3 by the Short-Circuit Current Contributions of Full-Sized Converters (acc. IEC 60909 resp. VDE 0102)

Full-size converter (FSC) systems are generation units connected to the grid via full-size converters. They are represented by ideal current sources ($\underline{Z}_\mathrm{i} \to \infty$). Their contributions to the short-circuit current are calculated with the current divider rule and taken into account in approximation by adding their absolute values to the initial symmetrical short-circuit current according to section 12.3.



Initial symmetrical short-circuit current[18][19][20]	$I''_\mathrm{kmax} = I''_\mathrm{koFSC} + I''_\mathrm{kFSC} = \frac{U_\mathrm{ers}}{\underline{Z}_\mathrm{k}} + \frac{1}{\underline{Z}_\mathrm{k}} \sum_{j=1}^{n} z_{ij} I_{\mathrm{qFSC}j}$
RMS value of the maximum source current in case of a 3-phase short-circuit of the full-size converter unit *j* (e. g. manufacturer information)	$I_{\mathrm{qFSC}j} = I_{\mathrm{kFSCmax}\,j}$
Absolute value of the elements of the node impedance matrix of the positive sequence[21]	$z_{ij} = \left\| \frac{\underline{U}_i}{\underline{I}_j} \right\|$ alle $\underline{I}_\mu = 0$ für $\mu \neq j$
Peak short-circuit current[22]	$i_\mathrm{p} = \kappa\sqrt{2}\, I''_\mathrm{koFSC} + \sqrt{2} I''_\mathrm{kFSC}$

For the calculation of I_b and I_k, reference is made to the details and requirements specified in IEC 60909. For the calculation of I_th refer to section 12.3.

12.5 Factors for the Calculation of Short-Circuit Currents

12.5.1 Factor *κ* for the Calculation of the Peak Short-Circuit Current i_p

Factor for the calculation of the peak short-circuit current[22] (Determination of κ by the methods a), b) and c) according to IEC 60909)	$\kappa = 1{,}02 + 0{,}98\,\mathrm{e}^{-3R/X}$

a) **Uniform ratio** R/X : At all short-circuit locations, the smallest ratio R/X of the network branches, that carry partial short-circuit currents at the nominal voltage corresponding to the short-circuit location, is applied.

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18) I''_koFSC is the maximum initial symmetrical short-circuit current without full-size converters (index oFSC) according to section 12.3.

19) For the calculation of the minimum initial symmetrical short-circuit current, the shares of the full-size converters are neglected.

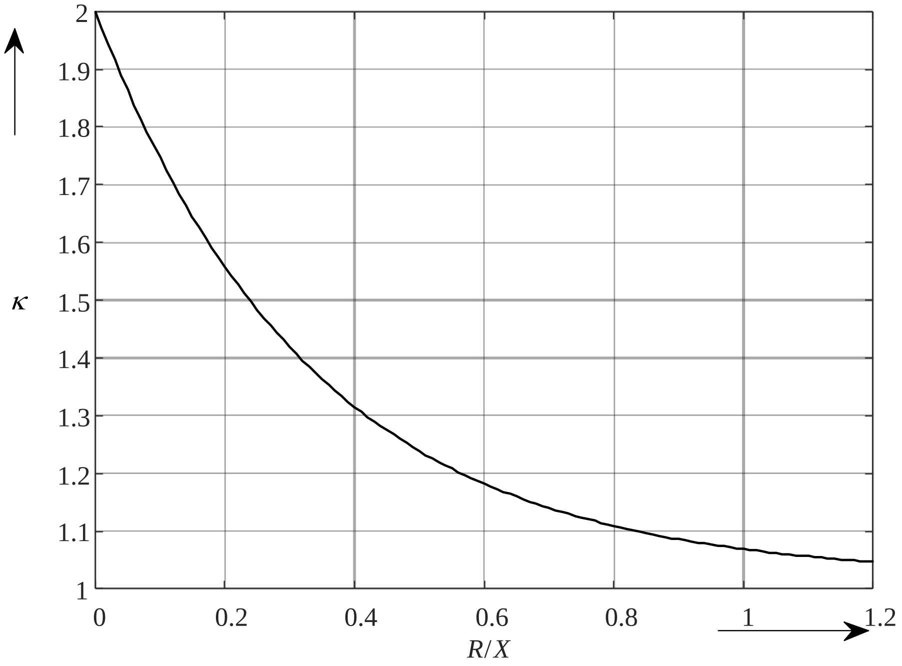
20) *i* identifies the node of the short circuit and *j* the nodes, where full-sized converters are connected to.

21) The determination results e.g. by calculating the voltage at node *i* in case of an exclusive feed-in of a node current $\underline{I}_j$ at node *j* of the short-circuit and source free network.

22) For further details and requirements please refer to IEC 60909.

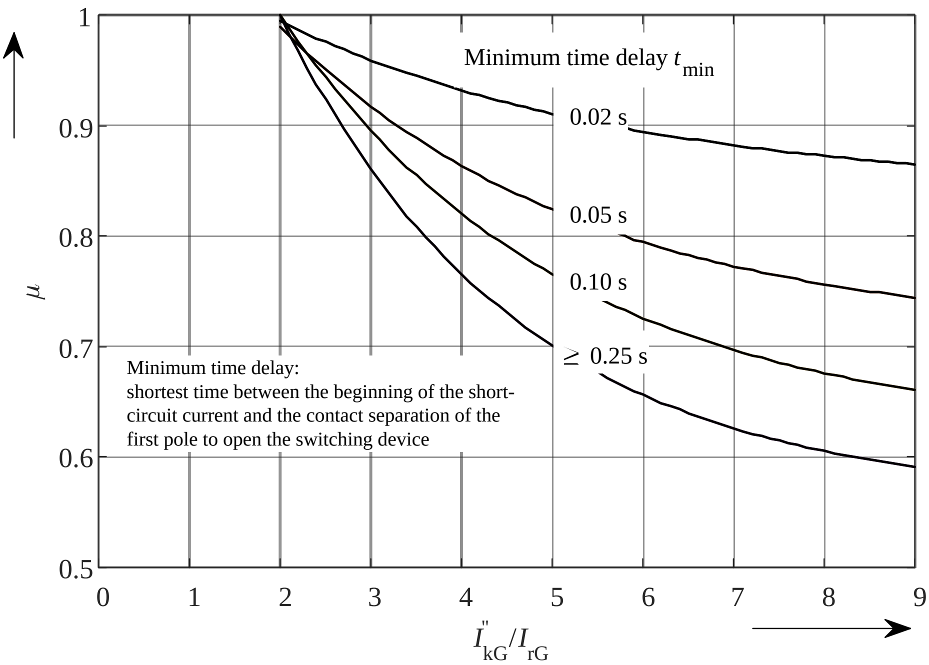
b) **Ratio R/X at the short-circuit location**: For the ratio R/X, the ratio R_k/X_k of the short-circuit impedance $\underline{Z}_k$ at the short-circuit location is used: $R/X = R_k/X_k$. To cover inaccuracies within the network reduction to a single short-circuit impedance, the factor $\kappa_{(b)} = 1{,}15 \cdot \kappa$ is applied, if $R_k/X_k \geq 0{,}3$, else $\kappa_{(b)} = \kappa$.

c) **Equivalent frequency f_c**: Determination of the short-circuit impedance at the short-circuit location $\underline{Z}_c = R_c + jX_c$ for the equivalent frequency $f_c = 20$ Hz (at nominal line frequency $f = 50$ Hz). It applies: $R/X = (R_c/X_c)\cdot(f_c/f)$.



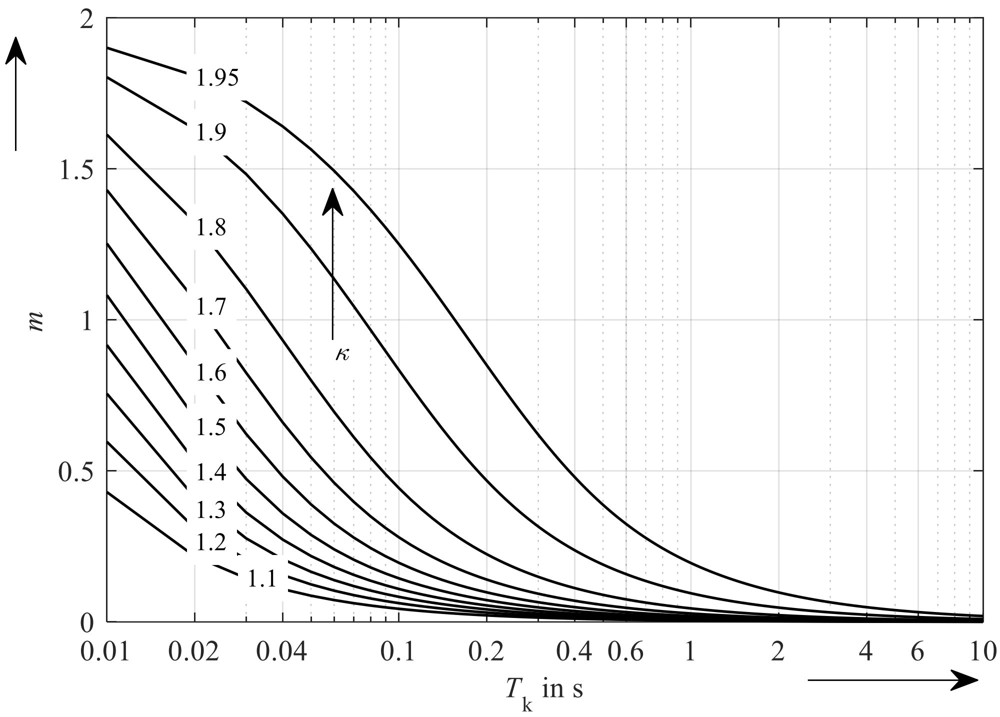
Factor κ for the calculation of the peak short-circuit current.

12.5.2 Factor μ for the Calculation of the Sym. Breaking Current I_b

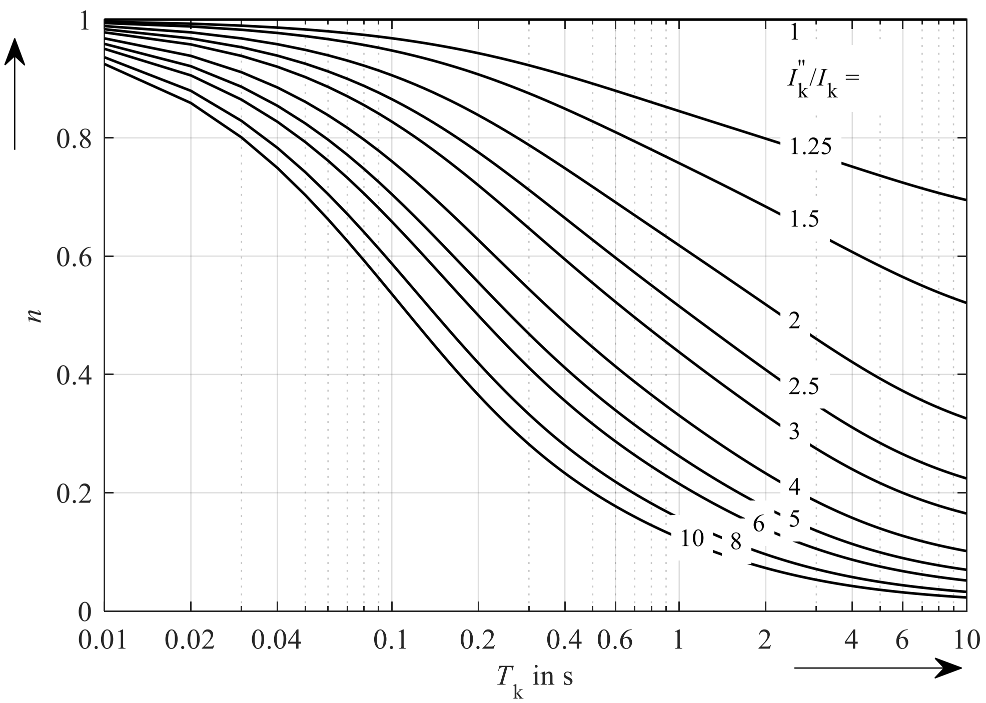


Factor μ for the calculation of the symmetrical breaking current in case of a near-to-generator single-fed three-phase short-circuit

12.5.3 Factors *m* and *n* for the Calculation of the Thermal Equivalent Short-Circuit Current I_{th}



Factor *m* for the consideration of the heat effect of the DC component of the short-circuit current



Factor *n* for the consideration of the heat effect of the AC component of the short-circuit current

12.6 Consideration of Impedance Correction Factors (according to IEC 60909 and VDE 0102)

Impedance correction factors for calculating the maximum short-circuit currents[23)]	
The short-circuit impedance $\underline{Z}_k$ at the short-circuit location should be calculated with corrected impedances. The defined value of the equivalent voltage source at the short-circuit location is in many cases not sufficient to calculate the initial symmetrical short-circuit currents and the partial initial short-circuit currents within a fault limit of $-5\,\%$. Therefore, the implementation of impedance correction factors is necessary.	
Two-winding transformers[24)]	$\underline{Z}_{TK} = K_T\,\underline{Z}_T$
	$K_T = 0{,}95\dfrac{c_{max}}{1+0{,}6\,x_T}$
Three-winding transformers	$\underline{Z}_{HVMV\,K} = K_{THVMV}\,\underline{Z}_{THVMV}$ $\underline{Z}_{HVLV\,K} = K_{THVLV}\,\underline{Z}_{THVLV}$ $\underline{Z}'_{MVLV\,K} = K_{TMVLV}\,\underline{Z}'_{TMVLV}$
	$K_{THVMV} = 0{,}95\dfrac{c_{max}}{1+0{,}6\,x_{THVMV}}$ $K_{THVLV} = 0{,}95\dfrac{c_{max}}{1+0{,}6\,x_{THVLV}}$ $K_{TMVLV} = 0{,}95\dfrac{c_{max}}{1+0{,}6\,x_{TMVLV}}$
Synchronous machines[25)]	$\underline{Z}_{GK} = K_G\,\underline{Z}_G$
	$K_G = \dfrac{U_{nN}}{U_{rG}}\dfrac{c_{max}}{1+x''_d\sqrt{1-\cos^2\varphi_{rG}}}$
Power station units with on-load tap changer[26)]	$\underline{Z}_{SK} = K_S\left(\ddot{u}^2\,\underline{Z}_G + \underline{Z}_{THV}\right)$
	$K_S = \dfrac{U_{nN}^2}{U_{rG}^2}\dfrac{U_{rTLV}^2}{U_{rTHV}^2}\dfrac{c_{max}}{1+\left\lvert x''_d - x_T\right\rvert\sqrt{1-\cos^2\varphi_{rG}}}$
Power station units without on-load tap changer[26)]	$\underline{Z}_{SOK} = K_{SO}\left(\ddot{u}^2\,\underline{Z}_G + \underline{Z}_{THV}\right)$
	$K_{SO} = \dfrac{U_{nN}}{U_{rG}\left(1+p_G\right)}\dfrac{U_{rTLV}}{U_{rTHV}}\dfrac{\left(1\pm p_T\right)c_{max}}{1+x''_d\sqrt{1-\cos^2\varphi_{rG}}}$

23) In case of unbalanced short-circuits, the impedance correction factors shall be applied to the negative- and the zero-sequence system. Impedances between a neutral point and ground shall be introduced without correction factor.

24) c_{max} is related to the nominal voltage of the network connected to the low-voltage side of the network transformer. The impedance correction is not valid for unit transformers and wind power station units.

25) Not to be used, if connected via a unit transformer.

26) To be used in case of short-circuits on the high-voltage side of the unit transformer.

12.7 Calculation with Kirchhoff's Laws

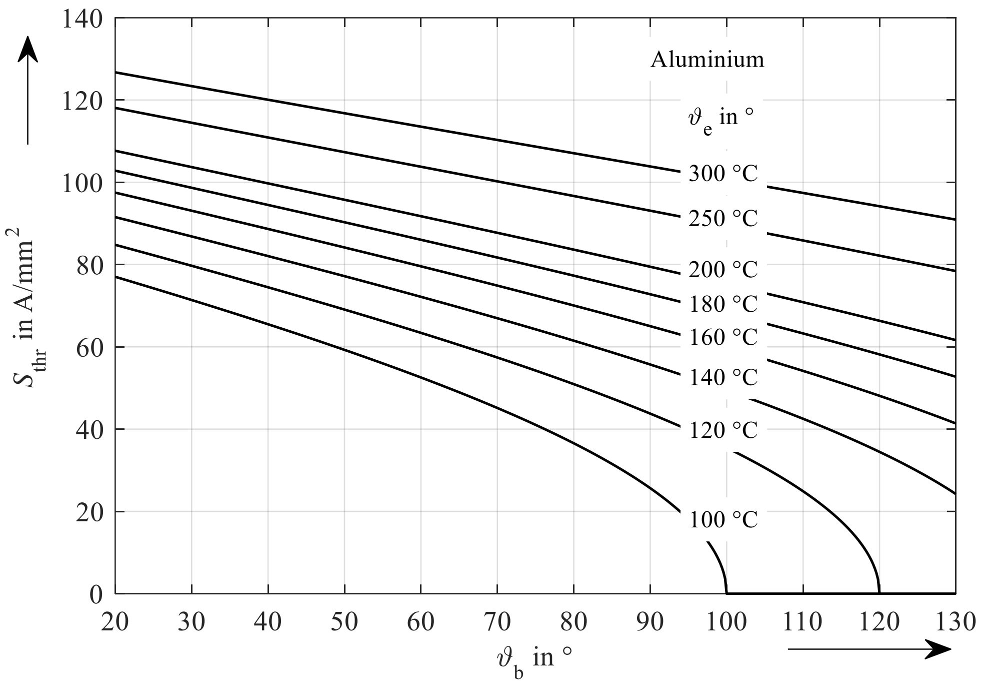
Calculation depends on operating point, exact calculation of the initial symmetrical short-circuit current $\underline{I}''_{\mathrm{k}}$ by magnitude and phase angle using the superposition method	
Calculation of partial short-circuit currents	$\underline{I}''_{\mathrm{kG}} = \dfrac{\underline{U}''}{\underline{Z}_{\mathrm{ers\,G}}}$, $\underline{I}''_{\mathrm{kN}} = \dfrac{\underline{U}_{\mathrm{N}}}{\underline{Z}_{\mathrm{ers\,N}}}$, …
Calculation of the exact initial symmetrical short-circuit current	$\underline{I}''_{\mathrm{k}} = \underline{I}''_{\mathrm{kG}} + \underline{I}''_{\mathrm{kN}} + \ldots$

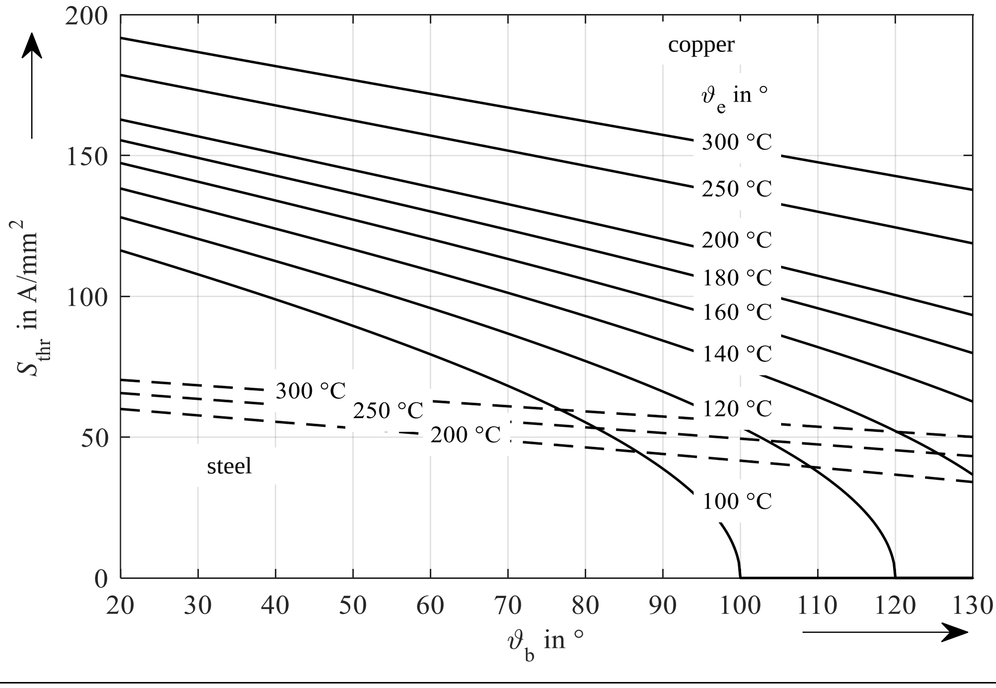
12.8 Thermal Short-Circuit Strength

DC resistance at ϑ_{20} κ_{20}: Specific conductivity at 20 C l: Conductor length A: Conductor cross-section	$R_0 = \dfrac{l}{\kappa_{20}\,A}$
DC resistance at ϑ_{x} α_{20}: Temperature coefficient of resistance at 20 C	$R = R_0\left(1+\alpha_{20}\left(\vartheta_{\mathrm{x}} - \vartheta_{20}\right)\right)$
Thermal equivalent short-circuit current	$I_{\mathrm{th}} = I''_{\mathrm{k}}\sqrt{m+n} = \sqrt{\dfrac{1}{T_{\mathrm{k}}}\int_0^{T_{\mathrm{k}}} i_{\mathrm{k}}^2(t)\,\mathrm{d}t}$
Duration of the short-circuit current (=sum of short-circuit durations in case of successive short circuits with short pauses)	$T_{\mathrm{k}} = \sum_{i=1}^{N_{\mathrm{k}}} T_{\mathrm{k}i}$
Determination of thermal short-time strength of electrical machines, transformers, transducers, reactors and switching devices	$I_{\mathrm{th}} \le I_{\mathrm{thr}}$ for $T_{\mathrm{k}} \le T_{\mathrm{kr}}$ $I_{\mathrm{th}} \le I_{\mathrm{thr}}\sqrt{\dfrac{T_{\mathrm{kr}}}{T_{\mathrm{k}}}}$ for $T_{\mathrm{k}} > T_{\mathrm{kr}}$
Thermal equivalent short-circuit current density ϑ_{b}: Conductor temperature at the beginning of a short-circuit ϑ_{e}: Conductor temperature at the end of a short-circuit	$S_{\mathrm{th}} = \dfrac{I_{\mathrm{th}}}{A} = \dfrac{I''_{\mathrm{k}}\sqrt{m+n}}{A}$ $= \sqrt{\dfrac{\kappa_{20}\,c_{\mathrm{p}}\,\rho}{\alpha_{20}\,T_{\mathrm{k}}}\ln\left(\dfrac{1+\alpha_{20}\left(\vartheta_{\mathrm{e}} - 20\,^{\circ}\mathrm{C}\right)}{1+\alpha_{20}\left(\vartheta_{\mathrm{b}} - 20\,^{\circ}\mathrm{C}\right)}\right)}$ c_{p}: Specific thermal capacity ρ: Density of conductor
Rated short-time withstand current density $S_{\mathrm{thr}} = S_{\mathrm{th}}$ calculated with: $\vartheta_{\mathrm{b}} = \vartheta_{\mathrm{b,max}}$: Maximum permissible operating temperature (unless otherwise known) $\vartheta_{\mathrm{e}} = \vartheta_{\mathrm{e,max}}$: Maximum permissible temperature in the event of a short-circuit $T_{\mathrm{k}} = T_{\mathrm{kr}}$: Rated duration of the short-circuit current (e.g. 1 s)	

Determination of thermal short-time strength of overhead line conductors, cables and busbars	$S_{th} \le S_{thr}\sqrt{\frac{T_{kr}}{T_k}}$

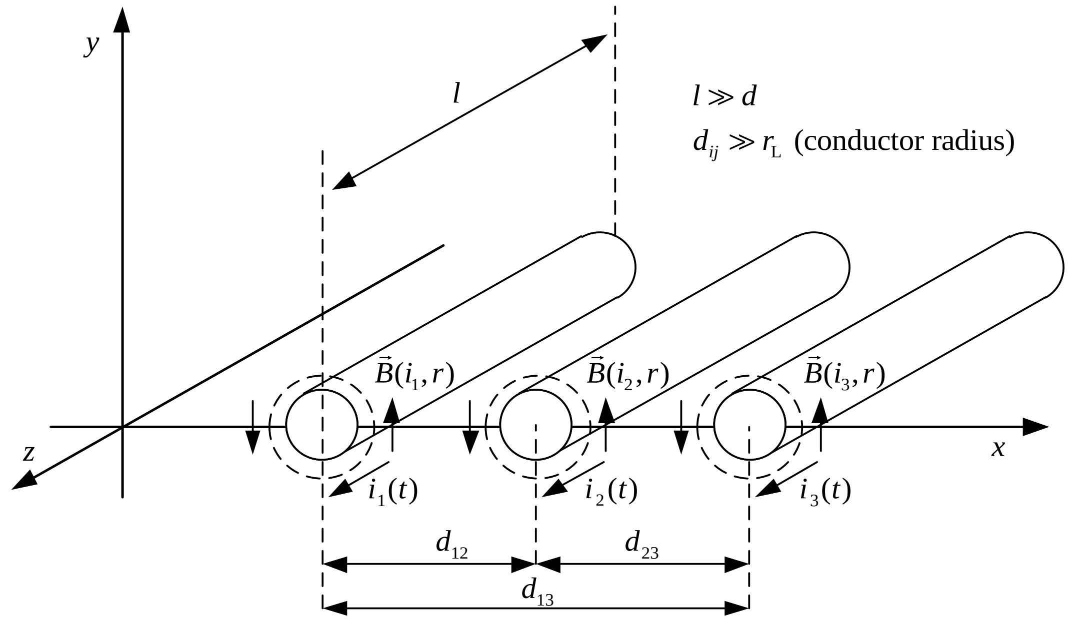
Rated short-time withstand current density depending on the conductor temperature at the beginning of the short-circuit ϑ_b and the conductor temperature at the end of the short-circuit ϑ_e for a rated duration of the short-circuit current $T_{kr} = 1\,s$ for aluminum, as wells copper and steel





12.9 Mechanical Short-Circuit Strength

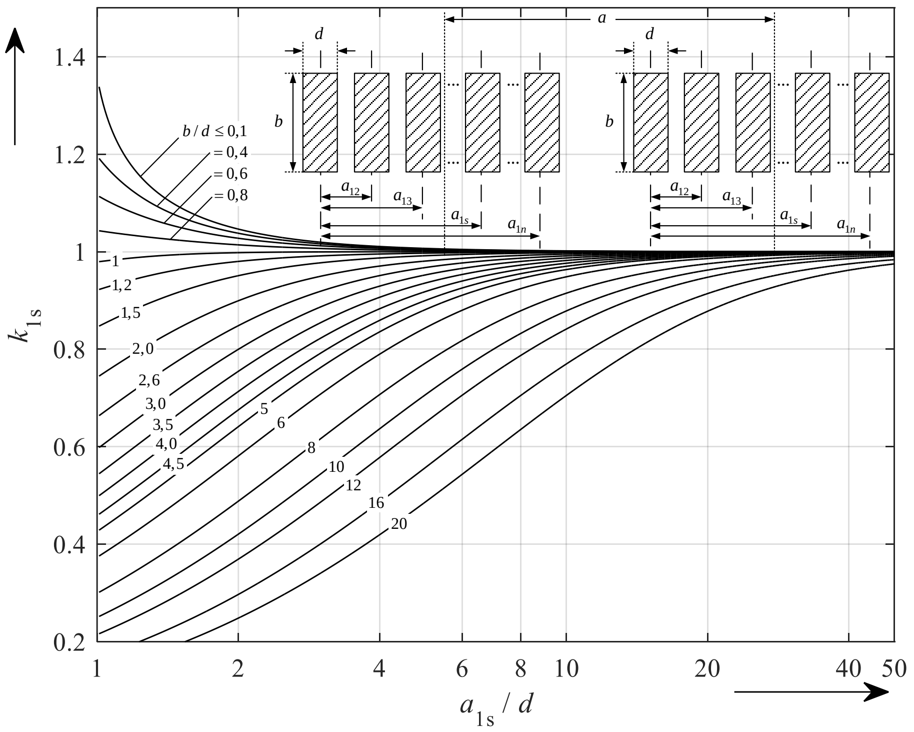
12.9.1 Determination of Magnetic Fields and Forces



Determination of field quantities	
Magnetic flux density	$\vec{B} = \mu_0 \, \mu_r \, \vec{H}$
Magnetic field strength	$\vec{H}(I, \vec{r}) = \frac{I_i}{2\pi \vec{r}}$
Magnetic force between conductors (Lorentz force, right-hand rule)	$\vec{F} = I \, (\vec{l} \times \vec{B})$
Determination of magnetic forces (three-phase system)	
Absolute value of the magnetic force between two conductors *i* and *j*	$F_{ij}(t) = \frac{\mu_0}{2\pi} \mu_r \, i_i(t) i_j(t) \frac{l}{d_{ij}}$

12.9.2 Maximum Forces Between Main Conductors and Sub-Conductors

Maximum main conductors force acc. IEC 60865-1 during a 3-phase short-circuit	
i_p : Peak short-circuit current l : Center-line distance between supports a_m : Effective distance between main conductors a : Center-line distance between conductors	$F_\mathrm{m} = \frac{\mu_0}{2\pi}\frac{\sqrt{3}}{2} i_\mathrm{p}^2 \frac{l}{a_\mathrm{m}}$
Main conductors consisting of single circular cross-sections	$a_\mathrm{m} = a$
Main conductors consisting of single rectangular cross-sections (factor $k_{1\mathrm{s}}$ see figure below)	$a_\mathrm{m} = \frac{a}{k_{1\mathrm{s}}}$
Maximum sub-conductors force acc. to IEC 60865-1 during a 3-phase short-circuit	
n : Number of sub-conductors l_s : Center-line distance between connecting pieces or between one connecting piece and the adjacent support	$F_\mathrm{s} = \frac{\mu_0}{2\pi}\left(\frac{i_\mathrm{p}}{n}\right)^2 \frac{l_\mathrm{s}}{a_\mathrm{s}}$ a_s : Effective distance between sub-conductors
Effective distance between sub-conductors with circular cross-sections	$\frac{1}{a_\mathrm{s}} = \sum_{i=2}^{n} \frac{1}{a_{1i}}$
Effective distance between sub-conductors with rectangular cross-sections (factor $k_{1\mathrm{s}}$ see figure below)	$\frac{1}{a_\mathrm{s}} = \sum_{i=2}^{n} \frac{k_{1i}}{a_{1i}}$



Factor $k_{1\mathrm{s}}$ for the calculation of the effective conductor distance

12.9.3 Calculation of Mechanical Stress

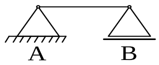
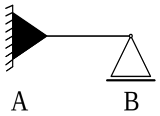
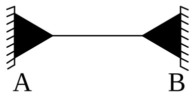
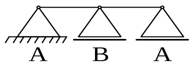
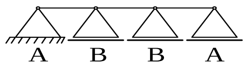
Bending stress caused by forces between main conductors	$\sigma_{\mathrm{m}} = V_{\sigma} V_{\mathrm{r}} \beta \frac{F_{\mathrm{m}} l}{8 Z}$

V_{σ} : Ratio of dynamic and static main conductor stress

V_{r} : Ratio of dynamic stress with unsuccessful three-phase automatic reclosing and dynamic stress with successful three-phase automatic reclosing

Z : Section modulus of main conductor in direction of the bending axis

β : Factor for main conductor stress (depends on type of beam and support)

Single-span beam	A and B simple supports		$\beta = 1{,}0$
	A: fixed support B: simple support		$\beta = 0{,}73$
	A and B fixed supports		$\beta = 0{,}5$
Continuous beam with equidistant simple supports	Two spans		$\beta = 0{,}73$
	Three or more spans		$\beta = 0{,}73$

Bending stress caused by the forces between the sub-conductors	$\sigma_{\mathrm{s}} = V_{\sigma\mathrm{s}} V_{\mathrm{rs}} \frac{F_{\mathrm{s}} l_{\mathrm{s}}}{16 Z_{\mathrm{s}}}$

The variables with the index s are defined analogously to the variables for the main conductors given above.

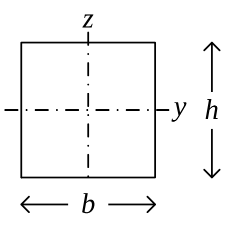
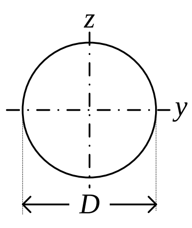
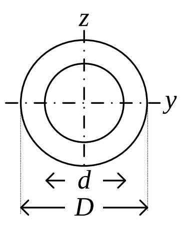
A conductor is mechanically short-circuit-proof, if:	$\sigma_{\mathrm{tot}} = \sigma_{\mathrm{m}} + \sigma_{\mathrm{s}} \le \sigma_{\mathrm{zul}} = q R_{\mathrm{p0,2}}$

σ_{tot} : Total conductor stress

$R_{\mathrm{p0,2}}$: Stress corresponding to the yield strength

q : Factor of plasticity (depends on the conductor profile and load)

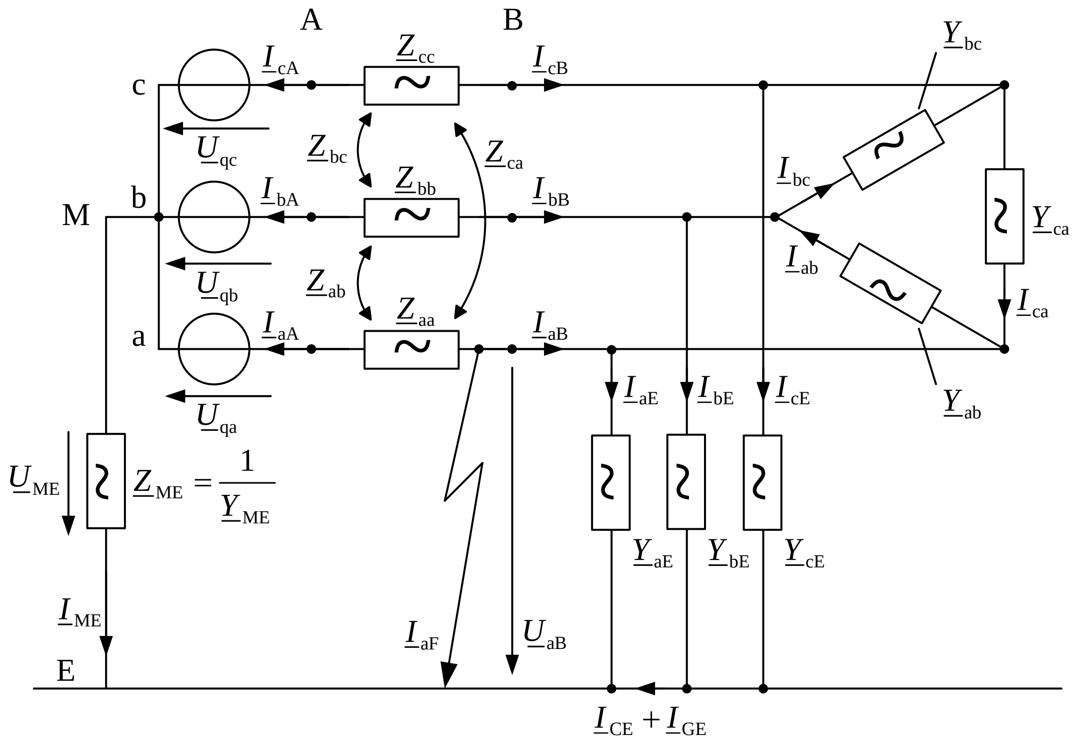
Section moduli *Z* are defined with respect to bending axis

rectangle	circle	circular ring
		
$Z_{\mathrm{y}} = \frac{bh^2}{6}, Z_{\mathrm{z}} = \frac{hb^2}{6}$	$Z = Z_{\mathrm{y}} = Z_{\mathrm{z}} = \frac{\pi}{32} D^3$	$Z = Z_{\mathrm{y}} = Z_{\mathrm{z}} = \frac{\pi}{32} \frac{D^4 - d^4}{D}$

13 Neutral Grounding

13.1 Equivalent Circuit for Single-Phase Line-to-Ground Faults

General equivalent circuit for neutral-point grounding



Assumption: Balanced three-phase system with $i,k = \mathrm{a,b,c}$

$$\underline{Y}_{i\mathrm{E}} = \underline{Y}_{\mathrm{E}} = \left(G'_{\mathrm{E}} + \mathrm{j}\omega C'_{\mathrm{E}}\right) l\ ,\ \underline{Y}_{ik} = \underline{Y} = \left(G' + \mathrm{j}\omega C'\right) l$$

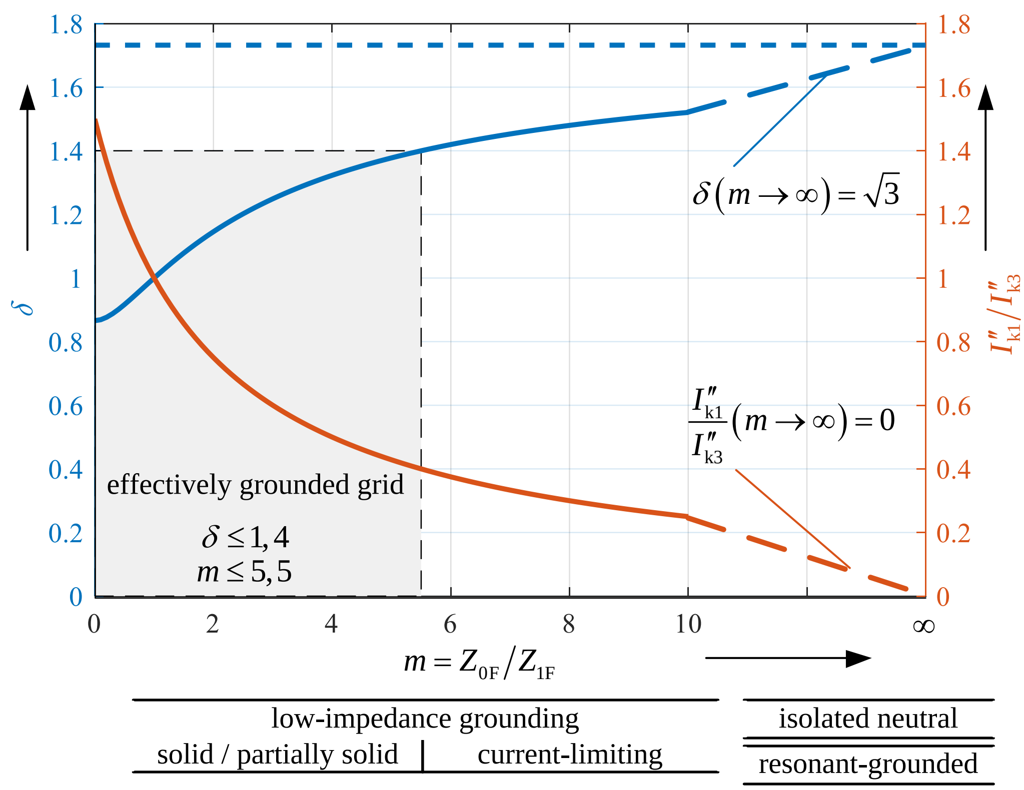
$$\underline{Z}_{ii} = \underline{Z}_{\mathrm{s}} = R_{\mathrm{s}} + \mathrm{j}X_{\mathrm{s}}\ ,\ \underline{Z}_{ik} = \underline{Z}_{\mathrm{g}} = R_{\mathrm{g}} + \mathrm{j}X_{\mathrm{g}}$$

$$\underline{Y}_{\mathrm{ME}} = \frac{1}{\underline{Z}_{\mathrm{ME}}} = \frac{1}{R_{\mathrm{ME}} + \mathrm{j}\omega L_{\mathrm{ME}}} \approx \frac{R_{\mathrm{ME}}}{R_{\mathrm{ME}}^2 + X_{\mathrm{ME}}^2} - \mathrm{j}\frac{1}{X_{\mathrm{ME}}} = G_{\mathrm{ME}} - \mathrm{j}\frac{1}{\omega L_{\mathrm{ME}}}$$

13.2 Currents and Voltages at Single-Phase Line-to-Ground Faults

Fault current at single-phase line-to-ground fault $\left(\underline{U}_{\mathrm{q1}} = \underline{U}_{\mathrm{qa}}\right)$	
Fault current at single-phase line-to-ground fault ($\underline{Z}_{\mathrm{1F}} = \underline{Z}_{\mathrm{2F}}$)	$\underline{I}_{\mathrm{aF}} = 3\underline{I}_{\mathrm{1F}} = -\underline{I}_{\mathrm{CE}} - \underline{I}_{\mathrm{GE}} - \underline{I}_{\mathrm{ME}} = 3\frac{\underline{U}_{\mathrm{q1}}}{\underline{Z}_{\mathrm{1F}} + \underline{Z}_{\mathrm{2F}} + \underline{Z}_{\mathrm{0F}}} = \frac{3}{2+\underline{m}}\frac{\underline{U}_{\mathrm{q1}}}{\underline{Z}_{\mathrm{1F}}}$
Capacitive ground fault current	$\underline{I}_{\mathrm{CE}} = \mathrm{j}\omega C_{\mathrm{E}}\left(\underline{U}_{\mathrm{bF}} + \underline{U}_{\mathrm{cF}}\right) = \mathrm{j}\omega C'_{\mathrm{E}}\, l\left(\underline{U}_{\mathrm{bF}} + \underline{U}_{\mathrm{cF}}\right)$
Conductive ground fault current	$\underline{I}_{\mathrm{GE}} = G_{\mathrm{E}}\left(\underline{U}_{\mathrm{bF}} + \underline{U}_{\mathrm{cF}}\right) = G'_{\mathrm{E}}\, l\left(\underline{U}_{\mathrm{bF}} + \underline{U}_{\mathrm{cF}}\right)$
Neutral-to-ground current	$\underline{I}_{\mathrm{ME}} = \underline{Y}_{\mathrm{ME}}\underline{U}_{\mathrm{ME}}$

Line-to-ground voltages of phases without faults	
Line-to-ground voltages of phases without faults $\underline{U}_{bF} = f_b(\underline{m})$, $\underline{U}_{cF} = f_c(\underline{m})$	$\underline{U}_{bF} = \frac{(\underline{a}^2 - \underline{a}) + (\underline{a}^2 - 1)\underline{m}}{2 + \underline{m}} \underline{U}_{q1} = -\frac{\sqrt{3}}{2}\left(\frac{\sqrt{3}\underline{m}}{2 + \underline{m}} + j\right)\underline{U}_{q1}$ $\underline{U}_{cF} = \frac{(\underline{a} - \underline{a}^2) + (\underline{a} - 1)\underline{m}}{2 + \underline{m}} \underline{U}_{q1} = -\frac{\sqrt{3}}{2}\left(\frac{\sqrt{3}\underline{m}}{2 + \underline{m}} - j\right)\underline{U}_{q1}$
Positive- / negative- / zero-sequence port impedances at short-circuit location	
Positive- and negative-sequence port impedances at short-circuit location	$\underline{Z}_{1F} = \underline{Z}_{2F} = \frac{1}{\frac{1}{\underline{Z}_1} + \underline{Y}_1} = \frac{1}{\frac{1}{\underline{Z}_s - \underline{Z}_g} + \underline{Y}_E + 3\underline{Y}}$
Zero-sequence port impedance at short-circuit location	$\underline{Z}_{0F} = \frac{1}{\frac{1}{\underline{Z}_0 + 3\underline{Z}_{ME}} + \underline{Y}_0} = \frac{1}{\frac{1}{\underline{Z}_s + 2\underline{Z}_g + 3\underline{Z}_{ME}} + \underline{Y}_E}$
Factor $\underline{m}$ (zero-sequence / positive-sequence port impedance ratio)	$\underline{m} = \frac{\underline{Z}_{0F}}{\underline{Z}_{1F}} \approx m \quad (\text{for } \angle\underline{Z}_{0F} \approx \angle\underline{Z}_{1F})$
Ground fault factor for pre-fault line-to-line voltage $U^b \approx \sqrt{3}U_{q1}$	
Ground fault factor for single-phase line-to-ground faults	$\delta = \frac{\max(U_{bF}, U_{cF})}{U^b/\sqrt{3}} \overset{\underline{m} \approx m}{=} \sqrt{3}\frac{\sqrt{m^2 + m + 1}}{2 + m}$

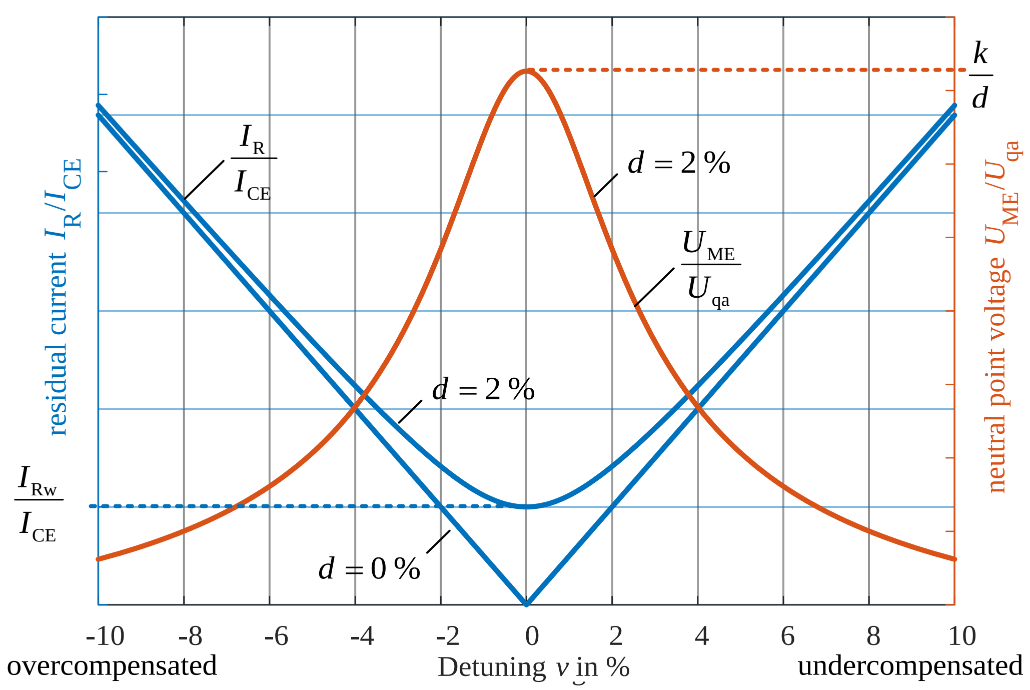


13.3 Isolated Neutral Point

$\lvert \underline{Z}_{ME} \rvert \to \infty$, $\lvert \underline{Z}_{1F} \rvert = \lvert \underline{Z}_{2F} \rvert \ll \lvert \underline{Z}_{0F} \rvert = \lvert 1/\underline{Y}_E \rvert$, $m \gg 1$	
Neutral-to-ground voltage	$\underline{U}_{ME} \approx -\underline{U}_{qa} = -\underline{U}_{q1}$
Fault current for single-phase line-to-ground fault = (negative) cap. ground fault current	$\underline{I}_{aF} = -\underline{I}_{CE} - \underline{I}_{GE} \approx -\underline{I}_{CE} = \mathrm{j}3\omega C_E \underline{U}_{q1}$
Line-to-ground voltages of phases without faults	$\underline{U}_{bF} \approx -\frac{\sqrt{3}}{2}\left(\sqrt{3}+\mathrm{j}\right)\underline{U}_{q1} = -\sqrt{3}\underline{U}_{q1}\,\mathrm{e}^{\mathrm{j}\frac{\pi}{6}}$ $\underline{U}_{cF} \approx -\frac{\sqrt{3}}{2}\left(\sqrt{3}-\mathrm{j}\right)\underline{U}_{q1} = -\sqrt{3}\underline{U}_{q1}\,\mathrm{e}^{-\mathrm{j}\frac{\pi}{6}}$
Ground fault factor	$\delta \approx \sqrt{3}$

13.4 Resonant-Grounded Neutral Point (RESPE)

$\lvert \underline{Z}_{ME} \rvert = f(v)$, $\lvert \underline{Z}_{1F} \rvert = \lvert \underline{Z}_{2F} \rvert \ll \lvert \underline{Z}_{0F} \rvert \approx \lvert 1/(\underline{Y}_{ME}/3+\underline{Y}_E) \rvert$, $m \gg 1$	
Neutral-to-ground voltage	$\underline{U}_{ME} \approx -\underline{U}_{qa} = -\underline{U}_{q1}$
Fault current for single-phase line-to-ground fault = ground fault current = residual current	$\underline{I}_{aF} = \underline{I}_R = -\underline{I}_{CE} - \underline{I}_{GE} - \underline{I}_{ME} = I_{Rw} + \mathrm{j} I_{Rb}$ $= \mathrm{j}\underline{I}_{CE}\left(d+\mathrm{j}v\right) = 3\omega C_E \left(d+\mathrm{j}v\right)\underline{U}_{q1}$
Detuning $v > 0 \to$ undercompensated $v < 0 \to$ overcompensated	$v = 1 - \frac{1}{3\omega^2 C_E L_{ME}}$
Attenuation	$d = \frac{G_{ME} + 3G_E}{3\omega C_E}$
Line-to-ground voltages of phases without faults	$\underline{U}_{bF} \approx -\sqrt{3}\underline{U}_{q1}\,\mathrm{e}^{\mathrm{j}\frac{\pi}{6}}$ $\underline{U}_{cF} \approx -\sqrt{3}\underline{U}_{q1}\,\mathrm{e}^{-\mathrm{j}\frac{\pi}{6}}$
Ground fault factor	$\delta \approx \sqrt{3}$
Neutral displacement voltage during undisturbed operation without ground fault Assumption: Unbalanced line-to-ground admittances $\underline{Y}_{aE} \neq \underline{Y}_{bE} \neq \underline{Y}_{cE}$	
Neutral-to-ground voltage	$\underline{U}_{ME} = \frac{-\underline{k}}{d+\mathrm{j}v}\underline{U}_{qa}$
Imbalance factor	$\underline{k} = \frac{\underline{Y}_{aE} + \underline{a}^2\underline{Y}_{bE} + \underline{a}\,\underline{Y}_{cE}}{3\omega C_E}$
Mean values of line-to-ground capacitances and conductances	$C_E = \frac{1}{3}\left(C_{aE} + C_{bE} + C_{cE}\right)$, $G_E = \frac{1}{3}\left(G_{aE} + G_{bE} + G_{cE}\right)$

Neutral displacement voltage during undisturbed operation and residual current in case of phase-to-ground faults in dependence of detuning


13.5 Low-Impedance-Grounded Neutral Point (NOSPE)

$\lvert \underline{Z}_{ME} \rvert = f\left(I''_{k1} < I''_{k1max}\right),\ \underline{Z}_{1F} = \underline{Z}_{2F} \approx \underline{Z}_1,\ \underline{Z}_{0F} \approx \underline{Z}_0 + 3\underline{Z}_{ME}$ $\underline{m} \approx 3\ldots5,5$ solidly grounded, partially solidly grounded, (HV and EHV grids) $\underline{m} > 4$ current-limiting grounding (10 – 110-kV cable grids)	
Neutral-to-ground voltage	$\underline{U}_{ME} \approx -\underline{U}_{qa} + \underline{Z}_s \underline{I}_{aF}$
Fault current for single-phase line-to-ground fault = line-to-ground short-circuit current	$\underline{I}_{aF} = \underline{I}''_{k1} = \frac{3}{2+\underline{m}} \frac{\underline{U}_{q1}}{\underline{Z}_{1F}} = \frac{3}{2+\underline{m}} \underline{I}''_{k3}$ with $\underline{U}_{q1} = c\frac{U_{nN}}{\sqrt{3}}$ (voltage factor c, see Section 12.3)
Line-to-ground voltages of phases without faults (see Section 13.2)	$\underline{U}_{bF} = f_b(\underline{m}),\ \underline{U}_{cF} = f_c(\underline{m})$ with $\underline{m} \approx \frac{\underline{Z}_0 + 3\underline{Z}_{ME}}{\underline{Z}_1}$
Ground fault factor	$\delta \le 1,4$ effectively grounded solidly / partially solidly grounding $\delta \approx 1,4\ldots\sqrt{3}$ **not** effectively grounded current-limiting grounding

14 Thermodynamics

Heat flow and quantity of heat	$P_{\text{th}}(t) = \frac{\mathrm{d}Q_{\text{th}}(t)}{\mathrm{d}t} = \dot{Q}_{\text{th}}(t)$
Law of conservation of energy (c_{p} : specific heat capacity)	$m c_{\text{p}} \Delta\vartheta = \Delta Q_{\text{th}} = Q_{\text{th,zu}} - Q_{\text{th,ab}}$
Thermal resistance for heat conduction (λ : thermal conductivity)	$R_{\text{th}} = \frac{\Delta\vartheta}{P_{\text{th}}} = \frac{\Delta x}{\lambda A}$
Thermal resistance for convection (α : heat transfer coefficient)	$R_{\text{th}} = \frac{\Delta\vartheta}{P_{\text{th}}} = \frac{1}{\alpha A}$
Analogy between thermal and electrical quantities	
Quantity of heat in J = Ws	Charge in C
Heat flow in W	Current in A
Difference in temperature in K	Voltage in V
Thermal capacity in J/K	Capacitance in F
Thermal resistance in K/W	Resistance in Ω
Thermal conductivity in W/(m·K)	Electrical conductivity in S/m

15 Wind Energy

Wind energy	$W_{\text{kin Wind}} = \frac{1}{2} m_{\text{Air}} v_{\text{Wind}}^2$
Wind power	$P_{\text{Wind}} = \frac{\mathrm{d}W_{\text{kin Wind}}}{\mathrm{d}t} = \frac{1}{2} \rho_{\text{Air}} A_{\text{Rotor}} v_{\text{Wind}}^3$
Rotor power (c_{p} : power coefficient)	$P_{\text{Rotor}} = \frac{1}{2} \rho_{\text{Air}} A_{\text{Rotor}} v_{\text{Wind}}^3 c_{\text{P}}(\lambda, \beta) = P_{\text{Wind}} c_{\text{P}}(\lambda, \beta)$
Tip speed ratio (n: rotational speed, R: radius of rotor)	$\lambda = \frac{v_{\text{Tip}}}{v_{\text{Wind}}} = \frac{2\pi n R}{v_{\text{Wind}}}$
Generator power (η_{ges} : total efficiency)	$P_{\text{Generator}} = P_{\text{Rotor}} \eta_{\text{ges}}$
Approximation of c_{p} with plant-specific constants c_i	$c_{\text{P}}(\lambda, \beta) = c_1 \left(\frac{c_2}{\lambda_1} - c_3\beta - c_4\beta^{c_5} - c_6 \right) \mathrm{e}^{-\frac{c_7}{\lambda_1}}$ with $\frac{1}{\lambda_1} = \frac{1}{\lambda + c_8\beta} - \frac{c_9}{\beta^3 + 1}$

16 Energy Economics

Relationship between energy and power	$P(t)=\frac{\mathrm{d}W(t)}{\mathrm{d}t}$ resp. $\Delta W=W(t_1)-W(t_0)$ $=\int_{t_0}^{t_1} P(t)\mathrm{d}t$
Average power	$P_\mathrm{m}=\frac{1}{T_\mathrm{N}}\int_0^{T_\mathrm{N}} P(t)\,\mathrm{d}t$
Efficiency	$\eta=\frac{\text{output power}}{\text{input power}}=\frac{P_\mathrm{ab}}{P_\mathrm{zu}}=\frac{P_\mathrm{zu}-P_\mathrm{V}}{P_\mathrm{zu}}$
Load factor m and utilization time T_m	$m=\frac{T_\mathrm{m}}{T_\mathrm{N}}=\frac{\int_0^{T_\mathrm{N}} P(t)/P_\mathrm{max}\,\mathrm{d}t}{T_\mathrm{N}}=\frac{W}{P_\mathrm{max}\,T_\mathrm{N}}=\frac{P_\mathrm{m}}{P_\mathrm{max}}$
Energy loss W_V and utilization time of power losses T_V	$W_\mathrm{V}=\vartheta_\mathrm{W}\,P_\mathrm{V\,max}\,T_\mathrm{N}=\int_0^{T_\mathrm{N}} P_\mathrm{V}(t)\mathrm{d}t$ $=P_\mathrm{V\,m}\,T_\mathrm{N}=P_\mathrm{V\,max}\,T_\mathrm{V}$
Loss factor	$\vartheta_\mathrm{W}=\frac{W_\mathrm{V}}{P_\mathrm{V\,max}\,T_\mathrm{N}}=\frac{\int_0^{T_\mathrm{N}} P_\mathrm{V}(t)\mathrm{d}t}{P_\mathrm{V\,max}\,T_\mathrm{N}}=\frac{P_\mathrm{V\,m}}{P_\mathrm{V\,max}}=\frac{T_\mathrm{V}}{T_\mathrm{N}}$ $\overset{U\approx\text{const.}}{=}\frac{R\int_0^{T_\mathrm{N}} I^2(t)\mathrm{d}t}{R\,I_\mathrm{max}^2\,T_\mathrm{N}}$ $\overset{I^2\sim S^2}{=}\frac{\int_0^{T_\mathrm{N}} S^2(t)\mathrm{d}t}{S_\mathrm{max}^2\,T_\mathrm{N}}=\frac{\int_0^{T_\mathrm{N}} (S(t)/S_\mathrm{max})^2\,\mathrm{d}t}{T_\mathrm{N}}$
Demand factor g (k: number of households)	$g(k)=\frac{p_\mathrm{SLA}(k)}{p_\mathrm{S}}\Leftrightarrow p_\mathrm{SLA}(k)=g(k)\,p_\mathrm{S}$ Peak load share p_SLA Connected load of household p_S
Approximation of demand factor for residential areas	$g(k)=g_\infty+(1-g_\infty)k^{-\frac{3}{4}}$ Limit for a high number of households g_∞

17 Annex

17.1 Selection of SI Base Units

Quantity	Symbol	Name	Unit symbol
Length	l	Meter	m
Mass	m	Kilogram	kg
Time	t	Second	s
Electric current	I	Ampere	A
Temperature	T	Kelvin	K

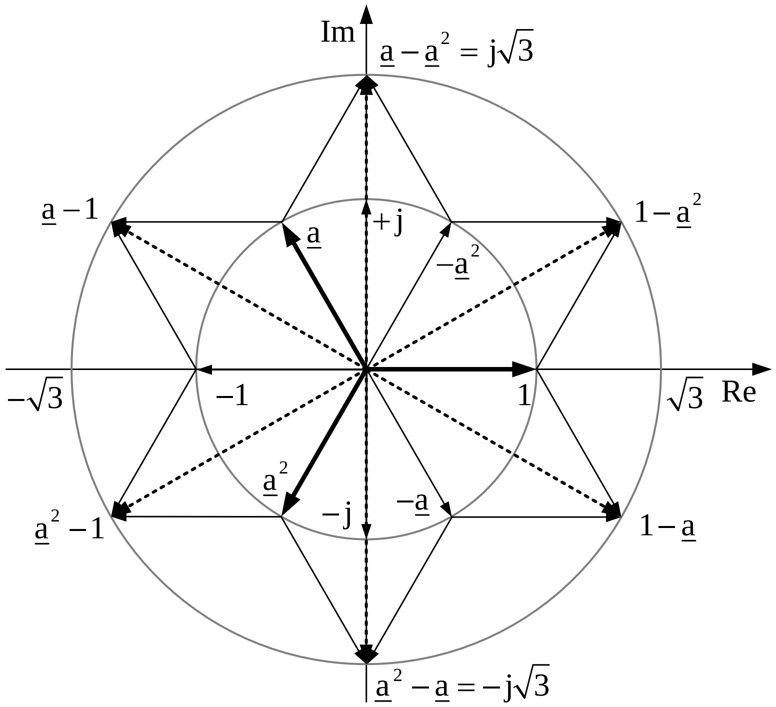
17.2 Selection of Derived Units

Parameter	Symbol	Unit	Unit symbol
Energy	W, Q	Joule	J
Density	ρ	Kilogram/ cubic meter	$\mathrm{kg/m^3}$
Torque	M	Newton meter	N m
Angle	α	Radiant	rad
Electric conductance	G	Siemens	S
Voltage	U	Volt	V
Electric current density	S	Ampere/square meter	$\mathrm{A/m^2}$
Electric resistance	R	Ohm	Ω
Frequency	f	Hertz	Hz
Speed	v	Meter/second	m/s
Inductance	L	Henry	H
Capacitance	C	Farad	F
Force	F	Newton	N
Charge	Q	Coulomb	C
Power	P	Watt	W
Magnetic field density	H	Ampere/meter	A/m
Magnetic flux density	B	Tesla	T
Moment of inertia	J	Kilogram · square meter	$\mathrm{kg\,m^2}$
Permeability	μ	Henry/Meter	H/m
Temperature	ϑ	Celsius	°C
Angular frequency	ω	Radiant/second	rad/s

17.3 Natural Constants and Mathematical Constants

Correlation of constants μ_0, ε_0 and c	$\mu_0 = 1/\varepsilon_0 c^2$	
Vacuum permeability μ_0	$\mu_0 = 1,25663...\cdot 10^{-6}$	$\mathrm{Vs/Am}$
Vacuum permittivity ε_0	$\varepsilon_0 = 8,85418...\cdot 10^{-12}$	$\mathrm{As/Vm}$
Speed of light c	$c = 299\,792\,458$	$\mathrm{m/s}$
Euler's constant e	$e = 2,71828...$	

17.4 Phasor Rotations with a and j



17.5 *n*-th Root of a Complex Number *G*

De Moivre's formula	$\sqrt[n]{\underline{G}} = \sqrt[n]{G}\cdot e^{\mathrm{j}\frac{\varphi_g + 2\pi k}{n}}$, for $k = 0, 1, \ldots, n\text{-}1$ for $n = 2$: $\sqrt{\underline{G}} = \pm\sqrt{G}\cdot e^{\mathrm{j}\frac{\varphi_g}{2}}$

17.6 Conversion Formulas for Hyperbolic and Exponential Functions

Theorem	Formula
Euler's formula	$e^{(\alpha+j\beta)} = \cosh(\alpha + j\beta) + \sinh(\alpha + j\beta) = e^{\alpha} \cdot e^{j\beta} = e^{\alpha}(\cos\beta + j\sin\beta)$
Hyperbolic functions with complex arguments	$\cosh(\alpha + j\beta) = \frac{1}{2}\left(e^{\alpha+j\beta} + e^{-(\alpha+j\beta)}\right) = \cosh(-(\alpha + j\beta))$
	$\sinh(\alpha + j\beta) = \frac{1}{2}\left(e^{\alpha+j\beta} - e^{-(\alpha+j\beta)}\right) = -\sinh(-(\alpha + j\beta))$
	$\sinh(\alpha + j\beta) = \sinh(\alpha)\cos(\beta) + j\cosh(\alpha)\sin(\beta)$
	$\cosh(\alpha + j\beta) = \cosh(\alpha)\cos(\beta) + j\sinh(\alpha)\sin(\beta)$
	$\cosh\alpha = \frac{1}{2}(e^{\alpha} + e^{-\alpha}) = \cos(j\alpha)$
	$\cosh(j\beta) = \frac{1}{2}(e^{j\beta} + e^{-j\beta}) = \cos(\beta)$
	$\sinh\alpha = \frac{1}{2}(e^{\alpha} - e^{-\alpha}) = -j\sin(j\alpha)$
	$\sinh(j\beta) = \frac{1}{2j}(e^{j\beta} - e^{-j\beta}) = j\sin(\beta)$
	$\cosh\alpha + \sinh\alpha = e^{\alpha}$
	$\cosh\alpha - \sinh\alpha = e^{-\alpha}$
	$\cosh^2\alpha - \sinh^2\alpha = 1$
	$\tanh\alpha = \frac{\sinh\alpha}{\cosh\alpha} = \frac{1}{\coth\alpha}$
De Moivre's formula for hyperbolic functions	$(\cosh\alpha \pm \sinh\alpha)^n = \cosh(n\alpha) \pm \sinh(n\alpha)$

17.7 Conversion Formulas for Trigonometric Functions

Trigonometric functions	$\sin\alpha = \frac{\text{opposite side}}{\text{hypotenuse}}$
	$\sin\alpha = \frac{1}{2j}\left(e^{j\alpha} - e^{-j\alpha}\right) = -\frac{1}{2j}\left(e^{-j\alpha} - e^{j\alpha}\right) = -\sin(-\alpha)$
	$\cos\alpha = \frac{\text{adjacent side}}{\text{hypotenuse}}$
	$\cos\alpha = \frac{1}{2}(e^{j\alpha} + e^{-j\alpha}) = \cos(-\alpha)$
	$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{1}{\cot\alpha} = \frac{\text{opposite side}}{\text{adjacent side}}$
	$\cos^2\alpha + \sin^2\alpha = 1$

Addition theorems	$\cos(\alpha \pm \beta) = \cos\alpha\ \cos\beta \mp \sin\alpha\ \sin\beta$
	$\sin(\alpha \pm \beta) = \sin\alpha\ \cos\beta \pm \cos\alpha\ \sin\beta$
	$\cos\left(\alpha \pm \frac{\pi}{2}\right) = \mp\sin\alpha = \pm\sin\left(-\alpha\right)$
	$\sin\left(\alpha \pm \frac{\pi}{2}\right) = \pm\cos\left(\alpha\right) = \pm\cos\left(-\alpha\right)$
	$\cos(\alpha - \frac{2\pi}{3}) = -\frac{1}{2}\cos\alpha + \frac{\sqrt{3}}{2}\sin\alpha$
	$\cos(\alpha + \frac{2\pi}{3}) = -\frac{1}{2}\cos\alpha - \frac{\sqrt{3}}{2}\sin\alpha$
	$\sin(\alpha - \frac{2\pi}{3}) = -\frac{1}{2}\sin\alpha - \frac{\sqrt{3}}{2}\cos\alpha$
	$\sin(\alpha + \frac{2\pi}{3}) = -\frac{1}{2}\sin\alpha + \frac{\sqrt{3}}{2}\cos\alpha$
	$\cos\alpha + \cos(\alpha - \frac{2\pi}{3}) + \cos(\alpha + \frac{2\pi}{3}) = 0$
	$\sin\alpha + \sin(\alpha - \frac{2\pi}{3}) + \sin(\alpha + \frac{2\pi}{3}) = 0$
	$\cos^2\alpha + \cos^2(\alpha - \frac{2\pi}{3}) + \cos^2(\alpha + \frac{2\pi}{3}) = \frac{3}{2}$
Multiplications	$\cos\alpha\ \cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$
	$\sin\alpha\ \sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$
	$\sin\alpha\ \cos\beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$
	$\cos\alpha\ \sin\beta = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$
	$\cos\alpha\ \cos(\alpha - \frac{2\pi}{3})\ \cos(\alpha + \frac{2\pi}{3}) = \frac{1}{4}\cos 3\alpha$
Multiples of an angle	$\sin(2\alpha) = 2\sin\alpha \cdot \cos\alpha$
	$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$
Law of sines	$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$ (The angles α, β, γ face a, b, c.)
Law of cosines	$a^2 = b^2 + c^2 - 2bc\cos\alpha$

17.8 Selection of Values of Trigonometric Functions

Angle	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
cos	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$