ANALYZING A NOISE SUPPRESSION CIRCUIT FOR IMPULSE DIVIDERS USING HAAR WAVELETS

Shuai Han, B. A. Chang, H. V. N. & Gockelbach, E.

Institute of Electric Power Systems, High Voltage Engineering Section,
Leibniz Universität Hannover, Germany

Abstract: Investigations on characteristics of divider and adjustments of its structural parameters in order to increase impulse voltages play a key role in high voltage measurements. When considering resistive switching impulse dividers, in particular, it is difficult to measure precisely the unit step response (USR) because of noise and large scale factor several thousands. One efficient solution is to mount a cylindrical metal shield at the lower end of the main resistor. However, powerful numerical divider simulations fail to compute the USR of dividers when the RC distributed circuit, to be composed by the main resistor and the cylindrical shield. This article presents approximation models of the RC distributed circuit of a 0.01 μs resistive switching impulse divider based on theoretical and waveform analysis, which can be used for USR simulations.

1. INTRODUCTION

Measurements of voltages and currents in high voltage engineering are difficult, especially in impulse fields, because in general both the highest value and the shape of signals should be recorded. This requires not only a specific measuring system but also an adequate recording system. Besides, the issue of suppression of noise arising from the structure of measuring systems and from the inhomogeneity between signals and measuring systems also plays a key role in precise high voltage measurements.

1.2 Potential dividers for high voltage impulse measurements

For impulse measurements, a few categories of dividers can be used, for example, resistive capacitive or clamped capacitive dividers [1]. Resistive dividers are only appropriate to accurately measure high voltage if the main resistance in high voltage arm is long enough so that the transfer behavior is not influenced by stray capacitances. In addition, adjustments on its structure are necessary, for example, mounting a cylindrical metal shield at the lower end of the main resistor, in order to change the influence of the stray capacitances. Resistive capacitive dividers operate in the same way like resistive capacitive dividers. In some cases, for preventing oscillations caused by a series resonance circuit originating from the combination between the inductance of the high voltage lead and the capacitance of the divider, the so-called clamped capacitive divider are exploited.

This article investigate theoretical and Wavelet-based analyses of a noise suppression circuit of the 0.01 μs resistive switching impulse divider, which has structural parameters 2000 μS for main resistor, 50 μF for measuring resistor, 500 μF for charging resistor, 2 cm diameter and 500 cm length for high voltage lead, placed 250 cm above the ground [2]. A noise suppression circuit is a distributed RC circuit, which is connected by a part of the main resistor and a cylindrical metal shield. Figure 1 shows these components.

1.2 Wavelet-based applications in circuit analysis

The principle of circuit analysis is based on solving differential and integral equations. There have been some powerful numerical tools used to deal with these issues. Since 1990s, wavelet-based methods, which relax to different order matrices such as Discrete, Coifman, Cohen, Daubechies, and Db-spline waves,... have been developed. In particular, Daubechies 8 and employed Daubechies wavelets in order to analyze successfully linear and nonlinear non-variant circuits [3]. However, Daubechies-based numerical solutions must be carried out separately for different types of integrals and in general, they are very complicated. This paper introduces a simple but effective solution, by using the Haar wavelet. As a conclusion quoted in [4], “This method is a powerful tool for solving differential and partial differential equations. The method with far less degree of freedom and with simpler CPU time provides better solutions than classical ones.”

2. ANALYSIS OF THE NOISE SUPPRESSION CIRCUIT FOR THE RESISTIVE DIVIDER

The resistance value of the main column of this divider is high and as a result, the noise factor is very high (0.006), and the Signal Noise (SN) ratio is small. Therefore, the precise measurement of the unit step response (USR) could not be done because of noises [5]. To overcome this problem, a cylindrical metal shield is mounted at the lower end of the main resistor, as shown in Fig. 1. The cylinder and a part of the main resistor establish a parallel capacitor, which, together with the resistor, creates a distributed circuit. In principle, the circuit works as an integral element, absorbing “external noise” and makes it possible to smoothen the USR, even with a large SN ratio.

However, the numerical divider simulation [5], which so far has successfully analyzed many dividers, fail to simulate the USR when the RC distributed circuit is attached. Furthermore, an approximation of a lossy LC distributed circuit cannot be simulated for a RC distributed circuit even in the EM field. Thus, it is necessary to analyze the approximation errors of the RC distributed circuit for USR simulations. Fig. 2 shows the main resistor divided into two parts. The left part (R1) is outside the shield, connected with the high voltage lead whereas the right part (R2) is inside the shield and connected with the low voltage arm. The distributed RC circuit formed by the resistor R2 and the shield appears on the right of the figure.
2.1 Theoretical Analysis

To analyze the distributed RC circuit, the replacement with a number of discrete elements is needed so that the approximating error is as small as possible. In theory, there are four types of discrete approximating circuits: R, OR, T, and M. For brevity and without loss of generality, this approximation by using discrete RC segments is illustrated here; the other two can be treated in the same way. In Fig. 1, a replacement of the distributed RC circuit by an array of discrete RC segments is carried out.

![Fig. 1 Approximation of the distributed RC circuit with discrete RC segments](image)

Consequently, the step response time ($\tau$) of a real circuit can be determined through the response time ($\tau_a$) of a normalized circuit:

$$\tau = \frac{\tau_a}{s} \quad (2)$$

where:

- $s = \frac{1}{\text{RC}}$
- $\tau_a$ is the response time of the real circuit

If the sum of the terminal constants $C_1, C_2, R_1,$ or $R_2$ in equation (1) is expressed under polynomial form like $C_1s^n$, then:

$$\tau_a = \frac{1}{\frac{s}{C_1s^n} + \frac{s}{C_2s^{n-1}} + \cdots + \frac{s}{C_2s^{n-1}}}$$

Equation (1) simplifies to:

$$\tau = \frac{\tau_a}{s} \quad (3)$$

This equation shows how the approximation error of the circuit in the frequency domain is reduced by the factor $s = \frac{1}{\text{RC}}$. Approximation errors of different approximating circuits are shown in Tab. 1.

![Fig. 2 Approximation circuit with operator (a) used in frequency domain](image)

<table>
<thead>
<tr>
<th>Approximation circuit</th>
<th>Input</th>
<th>Theoretical errors</th>
</tr>
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<tbody>
<tr>
<td>$R,$ $V$</td>
<td>$V$</td>
<td>$1/\text{RC}$</td>
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<tr>
<td>$R,$ $V$</td>
<td>$V$</td>
<td>$1/\text{RC}$</td>
</tr>
<tr>
<td>$T,$ $V$</td>
<td>$V$</td>
<td>$1/\text{RL}$</td>
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<tr>
<td>$T,$ $V$</td>
<td>$V$</td>
<td>$1/\text{RL}$</td>
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2.2 Wavelet-based solution

When determining the input source, the input source for a generator is the unit step voltage. However, due to the fact that the response $R$ has a very high value compared to that of the resistor $R_i$, thus a unit step current source would be considered as an input for the distributed RC circuit. On the other hand, the meaning of the wavelet $\Psi_{R_i}$ has a negligible value in comparison with that of $R_i$. Therefore, the value of $\Psi_{R_i}$ can be neglected. Hence, from equation (1), the UFR's of the circuit is given in frequency domain as:

$$\text{UFR}(s) = \frac{1}{\frac{s}{\text{RL}}} \quad (4)$$

2.2.1 Integral and derivative operators matrices using Haar Wavelet

The Haar wavelet function is defined in interval [0, 1] of real line. The matrix $\phi_{ii}$ of the wavelet is as follows:

$$\phi_{ii} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \quad (5)$$

Then, the fundamental size of the operational matrix has been introduced by the block pulse function $[7, 8]$ whose formula is defined as:

$$h(t) = \begin{cases} 0 & t < 0 \\ \frac{2^k}{\sqrt{N}} \cos \left( \frac{\pi}{2^k} t \right) & 0 \leq t \leq 2^k \pi \\ \frac{2^k}{\sqrt{N}} & \text{otherwise} \end{cases} \quad (6)$$

The integral operational matrix of the block pulse function $\Phi_{ii} = [7, 8]$ is defined [9]:

$$\Phi_{ii} = \frac{1}{2} \int_{0}^{2^k \pi} r(t) \phi_{ii} dt \quad (7)$$

where $\phi_{ii}$ is the operational matrix of block pulse function for integration at discrete points and is a set of orthogonal functions. Similarly, the integration of orthogonal functions $h(t)$ is given in the same way:

$$\Phi_{ii} = \frac{1}{2} \int_{0}^{2^k \pi} h(t) \phi_{ii} dt \quad (8)$$

Since the block pulse function $\Phi_{ii}$ is the identity matrix with the appropriate order and $h(t)$ is also a set of orthogonal functions, equation (8) can be rewritten as:

$$\phi_{ii} = \Phi_{ii} h(t) \quad (9)$$

A combination of equations (7) and (8) yields:

$$\phi_{ii} = \Phi_{ii} h(t) \quad \Phi_{ii} h(t) = \Phi_{ii} h(t) \quad (10)$$
From equations (8) and (9), one can derive the integral matrix of the four wavelet matrix $H_0$ in short form as:

$$Q_n = H_n A_{1}^T = H_n A_{1} H^T$$

(11)

Similarly, the discrete matrix of $H$ can be expressed:

$$Q_n = H_n A_{2}^T = H_n A_{2} H^T$$

(12)

Let considering a square integrable function $y(t)$ defined on $[0,L]$ to:

$$y(t) = \sum_{j=0}^{n} c_j \phi_j (t)$$

(13)

de then the forward four wavelet transform (FWT) can be executed under the matrix form as:

$$C^T = Y^T H^T = Y^T H$$

(14)

and the inverse four wavelet transform (IWFT) is expressed:

$$Y^T = C^{-1} H$$

(15)

2.2.3. Application for solving the USR of the noise suppression circuit of the divider

The goal of this part is to determine $h(t)$ or $V(t)$ in one tab by using the wavelet tool. Rewriting equation (11) in wavelet domain for cascaded discrete $R_C$ elements leads to:

$$\left[ \begin{array}{c} f(n) \\ f(n) \\ f(n) \\ f(n) \\ f(n) \\ f(n) \\ f(n) \\ f(n) \\ f(n) \\ f(n) \end{array} \right] = \left[ \begin{array}{c} I_1 \Phi_1 \Phi_2 \Phi_3 \Phi_4 \Phi_5 \Phi_6 \Phi_7 \Phi_8 \Phi_9 \end{array} \right]$$

where $I$ is the input matrix.

The equivalent form of equation (14) in wavelet domain can be obtained by:

$$USB = H_{n+1} = \frac{1}{2^{n+1}} f(n)$$

(18)

After getting the result in wavelet domain, $h_{n+1}$ should be converted into time domain by using IWFT. Figures 7 and 8 show simulation results along with the normalized RC circuit at different MRA levels.

For example, with 50 discrete RC segments used, the theoretical error is $2.53\%$. However, even with a small number of discrete T-type circuits deployed, there is no approximation error. Fig 9 presents the result derived in case the T-type is used.

3. CONCLUSIONS

1. Simulation results of all types of discrete approximating circuits show a good agreement with the theoretical values shown in Tab 1.

2. T-type and π-type approximations are the best ones from the view points of the above results and in theory being verified by wavelet-based circuit analysis tools, facilitating bit-redundant divider simulations.

3. For further applications in circuit analysis, the authors recommend this tool in an appropriate way for a variety of circuits.

ACKNOWLEDGEMENT

This paper is the result of the project entitled “Developing the enhanced vectorial model of dividers in high voltage measurement using wavelets” (project code: 104.01.2007.20) sponsored by the Ho Chi Minh City University of Technology.

4. REFERENCES

