A Continuous Parameter High Frequency Model based on Travelling Waves for Transformer Diagnostic Purposes

Asghar Akbari²  Peter Werle¹  Hossein Borsi¹  Ernst Gockenbach¹

¹University of Hannover, Germany
Institute of Electric Power Systems
Division of High Voltage Engineering, Schering-Institute
Callinstr. 25 A, D-30167 Hannover, Germany

²K.N. Tousi University of Technology
Dept. of Electrical Engineering
Tehran, Iran

Akbari@mbox.si.uni-hannover.de

Abstract: In this contribution a new continuous parameter model for power transformers, which is based on the travelling wave theory is suggested. In this modeling approach a small partial length along the winding is considered and a continuous circuit model is used to describe the Partial Differential Equation (PDE) of the coil structure. This equation determines the characteristics of the voltage wave propagation along the winding.

The experimental procedure of the PDE parameter optimization using measurements, which have been performed in the laboratory, is presented. The suggested model has been used to calculate the sectional winding transfer functions, which are required for a PD localization. Furthermore the results of a PD evaluation on a distribution transformer based on the new suggested model are illustrated.

INTRODUCTION

Recent work of the authors with the goal of using sectional winding transfer functions (SWTF) of transformers for the localization of partial discharges (PD) have shown excellent results [1]. The new suggested method for the evaluation and localization of PD, is based on the knowledge of the SWTFs of the transformer. For this method the PD signals are measured at two points of transformer coil e.g. at the bushing and at the neutral. These signals are mapped at various points along the winding using SWTFs. For each point the two mapped signals are compared. At the actual PD-source the similarity of these two calculated signals is maximum.

However, the SWTFs are not directly measurable for an actual transformer, thus the modeling of the transformer characteristic in a relatively wide frequency range is needed in order to compute these transfer functions. Because of the complicated RLC circuit path between the PD origin and the decoupling points an appropriate model has to be used, which enables the determination of the transmission behavior of the coil for transient PD signals.

A substantial number of transformer models for various objectives and applications have been proposed to date but the modeling in high frequency range, which accurately describes the behavior of the transformer coil is still an unsolved problem. The reason therefore is that transformers have a complicated winding system whose parameters are not constant in a wide frequency range.

The problem of transformer modeling in high frequency ranges can be categorized into two classes: detailed internal winding models and terminal models. For models belonging to the first class a quantitative description of the internal voltages and currents is possible using a large interconnected network of capacitances and coupled inductances. They have the advantage of allowing access to internal points along the winding, while with the models belonging to the second class only a terminal description in time and frequency domain is attainable by means of complex equivalent circuits [2]. It is evident, that for the problem of PD localization and evaluation a detailed internal winding model is required.

For the goal of PD localization a detailed model obtained from the discretization of the transformer winding to RLC equivalent circuits, which simulate the disc units of the coil has been suggested before [3]. In this model the coupling between various RLC units is represented by mutual inductances between sections, which causes the number of parameters of the model to increase. The parameter estimation of the model by means of comparing the model output and the measured quantities from the actual transformer even using the new soft computing techniques such as genetic algorithms is still very cumbersome.

In this contribution a new method of accessing the transient voltage behavior along the winding is suggested using the travelling wave theory. The base of this idea is the calculation of transient voltages at various points along the winding by means of solving Partial Differential Equations (PDE), which describe the frequency behavior of the coil. The solution thereof can be used for obtaining the SWTFs needed for diagnostic purposes.

CONTINUOUS PARAMETER MODEL

Some investigations have been performed earlier to compute the resonance frequencies of the coil by means of travelling wave theory [4]. This work has shown acceptable results for cylindrical and disc-type coils.

In this paper the travelling wave theory is used for the computation of SWTFs needed for transformer diagnostics. The continuous parameter model, which is based on travelling wave theory, considers a small differential length dx along the winding and a circuit model as presented in Figure 1 is assumed for this small part of the winding. The circuit constants per unit length of the winding are used for obtaining the PDE, which describes the characteristics of the travelling voltage wave passing through the winding.
The voltage along the winding in this model $e(x,t)$ is a function of the time and the distance to the end of the coil. If this function is known the relation between the voltages of the transformer terminals and each point along the coil as they are needed for the calculation of the SWTFs can be computed easily.

$$e(x,\infty) = \frac{\sinh(\beta x)}{\sinh(\beta l)} E \quad \text{with} \quad \beta = \sqrt{Gr/(1+gr)} \tag{4}$$

Applying the above mentioned terminal conditions and the initial distribution as well as the final distribution for the grounded winding, a solution in the following form is obtained:

$$e(x,t) = \frac{\sinh(\beta x)}{\sinh(\beta l)} E + \sum_{s=0}^{\infty} A_s \exp(-\gamma_s t) \cos(\omega_s t) \sinh(\frac{s\pi x}{l}) \tag{5}$$

$A_s, \gamma_s$ and $\omega_s$ are the amplitude, damping factor and frequency of the $s^{th}$ harmonic of the voltage wave at the point $x$, which are calculated using the PDE constant coefficients and the length of the winding.

**CALCULATION OF PDE COEFFICIENTS**

A step voltage is applied to the bushing of one phase of the transformer while the neutral is grounded. If the response measurement is possible at least at one accessible point along the winding, this measured input-output data may be used for the PDE parameter optimization. The parameters are optimized if the output computed by (5) for the specified measurement point $x$ becomes as analogous as possible to the measured value. After obtaining the optimal parameters the voltage can be computed easily using (5) for each arbitrary point along the coil, which is not accessible for measurement in a real transformer. Furthermore this calculated input-output data can be used for SWTF calculation needed for a PD localization.

The important point in parameter optimization in this case is, that the search space is very complicated and only new search methods such as genetic algorithms in combination with the conventional optimization methods e.g. Sequential Quadratic Programming (SQP) can lead to a solution [6].

For these investigations a specially prepared 10kV/380V distribution transformer has been used, where access to 7 clamps along the winding is possible [1].

Figure 2 depicts the measured response, which has been scanned from the center-point of the coil while a step signal has been applied to the bushing whereas the neutral has been grounded. The model response, which is the solution of the PDE after optimizing its coefficients, is shown as well.

With the substitution of the optimized parameters in the PDE and solving it for the solution of various points along the winding the time responses for each given point $x$ can be computed.

**PARAMETER CALCULATION FOR A REAL TRANSFORMER**

However for a real transformer there is no accessible point along the winding, which can be used for a measurement in order to optimize the PDE parameters. Therefore the PDE has to be solved again for an ungrounded winding and the voltage can be measured through an impedance between the end of the
coil and the ground and this voltage can be used to optimize the parameters of the PDE by genetic algorithms.

Another opportunity when using this modeling approach for a real transformer is the mathematical calculation of the current at the grounded end of the coil and using a current probe to measure the ground current while a step voltage is applied to the bushing. The comparison of these two current signals can be used to optimize the PDE parameters.

For mathematical calculation of the current via the neutral the summation of $i_1$, $i_2$, and $i_3$ in Figure 1 for $x=0$ (neutral point) approximates the ground current for a grounded coil. Thus we have:

$$i(x,t) = i_1 + i_2 + i_3 = (G + C \frac{\partial}{\partial t}) \int e(x,t)dx$$

substituting $e(x,t)$ from (5) to (6), integrating and simplifying results in an equation for the ground current, in which the DC term of the current has been neglected.

$$i(t) = GE \sum_{s=1}^{3} \frac{A_s}{s \pi} e^{-\gamma_s t} \cos(\omega_s t) +$$

$$CE \sum_{s=1}^{3} \frac{A_s}{s \pi} e^{-\gamma_s t} [\gamma_s \cos(\omega_s t) + \omega_s \sin(\omega_s t)]$$

**EXPERIMENTAL RESULTS**

The calculated and measured voltages along the winding at some points other than $x=50\%\ l$, which has been used for parameter optimization, are compared in Figure 3 for the distribution transformer introduced before. The results show a good match in time domain. The experiment has been repeated using different measuring points for parameter optimization but each time only one measuring clamp has been used to optimize the PDE parameters. Similar results have been acquired.

Figure 4 presents the SWTFs from various points of the coil to the bushing calculated by the suggested modeling method and those measured directly by impulse response. The SWTFs look similar around the first resonance frequency of the coil (about 64 kHz). The reason therefore is that as the parameter optimization has been performed in the time domain and the first resonance frequency is dominant, thus the effect of frequencies beyond that play a small role in the optimization process.

For the aim of PD evaluation and localization the validity of the SWTFs even in a limited frequency range gives an acceptable result. In order to investigate this matter 50 PD signals have been generated in a needle plane electrode configuration immersed in oil and injected into various clamps along the winding. After decoupling the responses at the bushing and at the neutral, the localization has been performed as described in [1] using the SWTFs calculated by the
modeling as well as those by direct measurement. The results of the localization are presented in Table 1.

Table 1: Localization error in percent of tested PD-signals using measured SWTFS (E1%) and SWTFS calculated by model (E2%). 50 PD signals generated in oil and injected into different points of the coil (0%: neutral, 100%: bushing).

<table>
<thead>
<tr>
<th>PD location</th>
<th>12.5%</th>
<th>25%</th>
<th>37.5%</th>
<th>50%</th>
<th>62.5%</th>
<th>75%</th>
<th>87.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1%</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>E2%</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

E1 and E2 represent the localization error in percent of tested PD-signals if measured and calculated SWTFS are used respectively. The frequency domain has been selected from 40 to 90 kHz, where the calculated and measured transfer functions have a good match. The results show, that this method gives a good solution to the problem of PD localization for a real transformer.

CONCLUSION

For the aim of PD localization and evaluation a model is required, which enables the computation of transient voltages at the internal points along the transformer winding. Parameter estimation for the conventional detailed network models for transformer coils is still very cumbersome according to the high number of parameters. The continues parameter model based on the travelling wave theory enables the calculation of the transient voltages at the internal points of the transformer coil required for PD localization. The optimization of PDE coefficients is possible using at least a measurable voltage at a point along the winding or the current at the neutral for a grounded winding. Because the first resonance frequency is dominant in the response of the coil a better criteria for optimization has to be investigated in order the model as well as the coil response to be matched in a wide frequency range.

ACKNOWLEDGMENT

The authors like to thank the Alexander von Humboldt Foundation for granting the research fellowship of Dr. Akbari.

REFERENCES

   Localisation and evaluation of partial discharges on power transformers using sectional winding transfer functions

   A high frequency transformer model for the EMTP,

   High frequency transformer model for computation of sectional winding transfer functions used for partial discharge localisation

   Study of power Coil Resonance Phenomena based on Travelling Wave Theory
   IEEE transactions on Electrical insulation, Vol. 26 No. 3, June 1991

   Travelling waves on transmission systems
   Dover Publication, Inc., 1951

   A genetic algorithm for function optimization: A Matlab implementation
   ACM Transactions on Mathematical Software, 1996